NELSON SENIOR MATHS METHODS 12
FULLY WORKED SOLUTIONS

Chapter 5 Binomial distributions
Exercise 5.01 The Bernoulli distribution

Concepts and techniques

1 **b**, **c** and **e** each have two outcomes, one of which can be considered as a success and has a fixed probability

2 E $1-\frac{3}{30}=\frac{27}{30}=\frac{9}{10}$

3 E $\frac{2}{5}$, $p=\frac{1}{5}, q=\frac{4}{5}$ and $\sigma=\sqrt{p(1-p)}=\sqrt{\frac{1}{5}\times\frac{4}{5}}=\frac{2}{5}$

4 **a** Yes, as there are two possible outcomes.

b No, as there are more than two possible outcomes

c No, as there are more than two possible outcomes.

d Yes, as there are two possible outcomes.

e Yes, as there are two possible outcomes.

f No, as there are more than two possible outcomes.

g No, as there are more than two possible outcomes.

5 **a** P(a red onion) = 0.5

b P(a six) = $\frac{1}{8}$

c P(a sherbet) = $\frac{1}{4}$

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6 **a** $p = \frac{2}{5}, q = \frac{3}{5}, n = 1$ Variance $= \sigma^2 = pq = \frac{6}{25}$

b $q = \frac{1}{3}, p = \frac{2}{3}, n = 1$ Variance $= \sigma^2 = pq = \frac{2}{9}$

c $p = 0.6,\ q = 0.4,\ n = 1$ Variance $= \sigma^2 = pq = 0.24$

d $q = 0.3,\ p = 0.7, n = 1$ Variance $= \sigma^2 = pq = 0.21$

e $p = \frac{3}{4}, q = \frac{1}{4}, n = 1$ Variance $= \sigma^2 = pq = \frac{3}{16}$

7 **a** $q = 0.4,\ p = 0.6, n = 1$ Standard deviation $= \sigma = \sqrt{pq} = \sqrt{0.24} = 0.49$

b $p = \frac{2}{3}, q = \frac{1}{3}, n = 1$ Standard deviation $= \sigma = \sqrt{pq} = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$

c $q = \frac{24}{25}, p = \frac{1}{25}, n = 1$ Standard deviation $= \sigma = \sqrt{pq} = \sqrt{\frac{24}{625}} = \frac{2\sqrt{6}}{25}$

d $p = 0.2,\ q = 0.8, n = 1$ Standard deviation $= \sigma = \sqrt{pq} = \sqrt{0.16} = 0.4$

e $p = \frac{7}{9}, q = \frac{2}{9}, n = 1$ Standard deviation $= \sigma = \sqrt{pq} = \sqrt{\frac{14}{81}} = \frac{\sqrt{14}}{9}$

Reasoning and communication

8 $P(x = 0) = \frac{23}{25} \times \frac{23}{25} \times \frac{23}{25} \times \frac{23}{25} \times \frac{23}{25} = \frac{6\,436\,343}{9\,765\,625} \approx 0.659\,082$

or $P(x = 0) = \frac{{}^{92}C_5}{{}^{100}C_5} = 0.65908$

9 Two red, three green and one blue marble.

P(a red marble) $= \frac{1}{3}$.

Standard deviation $= \sigma = \sqrt{pq} = \sqrt{\frac{1}{3} \times \frac{2}{3}} = \frac{\sqrt{2}}{3}$

10 P(ace) $= \frac{4}{52} = \frac{1}{13}$.

$p = \frac{1}{13},\ q = \frac{12}{13}$ Variance $= \sigma^2 = pq = \frac{12}{169}$

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11 $P(x=0) = 0.8 \times 0.8 \times 0.8 \times 0.8 \times 0.8 = 0.8^5 = 0.32768$

12 200 = 20 misshaped + 180 normal

$$\begin{aligned} P(\text{at most } 1) &= P(x=0) + P(x=1) \\ &= \frac{{}^{20}C_0 \times {}^{180}C_{10}}{{}^{200}C_{10}} + \frac{{}^{20}C_1 \times {}^{180}C_9}{{}^{200}C_{10}} \\ &\approx \frac{1 \times 7.63 \times 10^{15}}{2.245 \times 10^{16}} + \frac{20 \times 4.4609 \times 10^{14}}{2.245 \times 10^{16}} \\ &= 0.339\,77 + 0.397\,40 \\ &\approx 0.737\,17 \end{aligned}$$

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Exercise 5.02 The geometric distribution

Concepts and techniques

1 D 'the number of digits I read beginning at a randomly selected point in a table of random digits until I find a 9'

This is failure, failure, ..., failure, success.

2 B because $(0.2)^4(0.8) = 0.001\,28$

3 D 0.0723, $P(X = 4) = (0.7)^4(0.3) = 0.07203$

4 B $P(X > 3) = 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3)$

$= 1 - [0.8 + 0.2 \times 0.8 + (0.2)^2 \times 0.8 + (0.2)^3 \times 0.8]$

$= 0.0016$

5 A $E(X) = \dfrac{1-p}{p} = \dfrac{1-0.8}{0.8} = 0.25$

6 **a** Yes, as failure, failure, success with the same probabilities.

b No, as the probabilities do not stay the same.

c Yes, as a sequence of q, q, ..., q, p with the same probabilities.

d Yes, as a sequence of B, B, ..., B, G with the same probabilities.

e Yes, as a sequence of N, N, ..., N, Y with the same probabilities.

7 $P(\text{exactly six rolls are required}) = \left(\dfrac{4}{6}\right)^5\left(\dfrac{2}{6}\right) = 0.0439$

8 $P(\text{H}) + P(\text{MH}) + P(\text{MMH}) = \dfrac{1}{3} + \dfrac{2}{3} \times \dfrac{1}{3} + \dfrac{2}{3} \times \dfrac{2}{3} \times \dfrac{1}{3} = \dfrac{9+6+4}{27} = \dfrac{19}{27} = 0.704$

9 **a** $P(X = 5) = 0.8^5 \times 0.2 = 0.0655$

b $P(X \leq 5) = 0.2 + 0.8^1 \times 0.2 + 0.8^2 \times 0.2 + 0.8^3 \times 0.2 + 0.8^4 \times 0.2 + 0.8^5 \times 0.2$

$= 0.2(1 + 0.8 + 0.8^2 + 0.8^3 + 0.8^4 + 0.8^5)$

$= 0.738$

c $E(\text{X}) = \dfrac{1-p}{p} = \dfrac{1-0.2}{0.2} = 4$

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10 $P(\text{underweight}) = \frac{3}{4}$

a $P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{1}{4} = \frac{3}{4}$

b $P(1 \leq X \leq 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$

$$= \frac{3}{4} \times \frac{1}{4} + \left(\frac{3}{4}\right)^2 \times \frac{1}{4} + \left(\frac{3}{4}\right)^3 \times \frac{1}{4} + \left(\frac{3}{4}\right)^4 \times \frac{1}{4} + \left(\frac{3}{4}\right)^5 \times \frac{1}{4}$$

$$= \frac{3}{4} \times \frac{1}{4}\left[1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \left(\frac{3}{4}\right)^4\right] = 0.572$$

11 $P(\text{hit his drive straight}) = 0.1$

a $P(4 \leq X \leq 14) = (0.9)^4(0.1) + (0.9)^5(0.1) + ... + (0.9)^{14}(0.1)$

$= (0.9)^4(0.1)\{1 + 0.9 + 0.9^2 + ... + (0.9)^{10}\}$

(Note: Geometric series)

$= (0.9)^4(0.1)\{6.861\ 894\}$

$= 0.45$

b $E(X) = \frac{1-p}{p} = \frac{1-0.1}{0.1} = 9$

12 $P(\text{correct}) = 0.1$

Y denotes the number of questions answered for the first correct answer.

a $P(Y = 7) = (0.9)^6(0.1) = 0.0531$

b $P(2 \leq Y \leq 8) = (0.9)(0.1) + (0.9)^2(0.1) + (0.9)^3(0.1) + (0.9)^4(0.1) + (0.9)^5(0.1)$

$+ (0.9)^6(0.1) + (0.9)^7(0.1)$

$= (0.9)(0.1)\{1 + (0.9)^1 + (0.9)^2 + (0.9)^3 + (0.9)^4 + (0.9)^5 + (0.9)^6\}$

$P(2 \leq Y \leq 8) = 0.4695$

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Reasoning and communication

13 $P(\text{even}) = 0.5$

a $P(\text{exactly 2}) = 0.5 \times 0.5 = 0.25$

b $P(\text{at least 2}) = P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$

$= 1 - 0.5 - 0.25 = 0.25$

c $P(\text{no more than 2}) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0.5 + 0.5 \times 0.5 + 0.5 \times 0.5 \times 0.5$

$= 0.875$

14 $P(\text{sinks putt}) = \frac{3}{4}$

$P(\text{qqqqp}) = \left(\frac{1}{4}\right)^4 \times \frac{3}{4} = 0.00293$

15 $P(\text{speed}) = 0.1$

$P(\text{qqqqqp}) = (0.9)^5 \times 0.1 = 0.059$

16 $P(\text{fraud}) = 0.25$

a $P(\text{qqqqq}) = 0.75^5 = 0.237$

b $E(X) = \frac{1-p}{p} = \frac{1-0.25}{0.25} = 3$

i.e., you would get away with two and the third one would be expected to be detected.

17 $P(\text{job}) = 0.75$

$P(\text{qqqp}) = (0.25)^3 0.75 \approx 0.0117$

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Exercise 5.03 The binomial distribution

Concepts and techniques

1 D 'the number of 9s in a randomly selected set of 10 digits from a table of random digits' is binomial as you know how many are 9s and how many are not 9s.

2 B The binomial distribution with $n = 6$ and $p = 0.5$.

3 E $4(\frac{5}{6})^3(\frac{1}{6})^1$ as ${}^4C_3 = 4$ and we want 3 successes $\frac{5}{6}$, and one failure $\frac{1}{6}$.

4 **b**, **c**, **e** and **g** as there are a fixed number of Bernoulli trials with the same probabilities of success

5 **a** $\binom{6}{4}(0.7)^4(0.3)^2 = 0.3241$

b $\binom{9}{3}(0.38)^3(0.62)^6 = 0.2618$

c $\binom{5}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3 = 0.2048$

d $\binom{8}{7}(0.25)^7(0.75)^1 = 0.0004$

e $\binom{10}{0}(0.09)^0(0.91)^{10} = 0.3894$

6 **a** $\binom{10}{2}(0.5)^2(0.5)^8$ $p = 0.5$, $q = 0.5$, $n = 10$, $x = 2$

b $\binom{20}{0}(0.85)^{20}$ $p = 0.15$, $q = 0.85$, $n = 20$, $x = 0$

c $\binom{15}{12}\left(\frac{3}{5}\right)^{12}\left(\frac{2}{5}\right)^3$ $p = \frac{3}{5}$, $q = \frac{2}{5}$, $n = 15$, $x = 12$

d $\binom{9}{8}(0.11)^8(0.89)$ $p = 0.11$, $q = 0.89$, $n = 9$, $x = 8$

e $\binom{7}{4}(0.25)^4(0.75)^3$ $p = 0.25$, $q = 0.75$, $n = 7$, $x = 4$

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7 $P(X = x) = \binom{7}{x}(0.8)^x(0.2)^{7-x}$.

a The number of trials = 7

b The probability of success in any trial is 0.8

c

x	0	1	2	3	4	5	6	7
$P(X = x)$	0.0000	0.0004	0.0043	0.0287	0.1147	0.2753	0.3670	0.2097

8 $P(Z = z) = \binom{7}{z}(0.15)^z(0.85)^{7-z}$.

a The number of trials is 7.

b The probability of success in any trial is 0.15.

c

z	0	1	2	3	4	5	6	7
$P(Z = z)$	0.3206	0.3960	0.2097	0.0617	0.0109	0.0012	0.0001	0.0000

9 $n = 7, x = 3, p = \frac{1}{6}, q = \frac{5}{6}$

$P(X = 3) = {}^7C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^4 = 0.0781$

Reasoning and communication

10 The value of p in question **7** is much higher than the value of p in question **8** and the probabilities of more successes in question **7** are much higher than the corresponding probabilities in question **8**. Similarly, the probabilities of more failures in question **7** are much lower than the corresponding probabilities in question **8**.

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Exercise 5.04 Using the binomial distribution

Concepts and techniques

1 D $P(X = 5) = {}^{6}C_{5}\left(\frac{5}{6}\right)^{5}\left(\frac{1}{6}\right)^{1} = 0.401\ 877$

2 B Binomial probability distribution with $n = 7$ and $p = \frac{2}{5}$.

$P(X \text{ is at least } 6) = P(X = 6) + P(X = 7)$

$= 0.0172 + 0.0016$

$= 0.0188$

3 D $p = 0.4$, $n = 5$, $P(\text{at most one}) = P(X = 0) + P(X = 1)$

$= 0.0778 + 0.2592$

$= 0.337$

4 D $n = 8$, $p = \frac{1}{3}$, $P(X \geq 1) = 1 - P(X = 0) = 1 - 0.039 = 0.9610$

5 B $n = 10$, $p = 0.75$, $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0.000\ 0010 + 0.000\ 0286 + 0.000\ 3862$

$= 0.000\ 4158$

6 $n = 7$, $p = 0.4$

a $P(X = 3) = 0.2903$

b $P(X \text{ is at least } 3) = P(X = 3) + P(X = 4) + \ldots + P(X = 7) = 1 - 0.419\ 904 = 0.5801$

c $P(X \text{ is more than } 5) = P(X = 6) + P(X = 7) = 1 - 0.998\ 115\ 84 = 0.018\ 8416$

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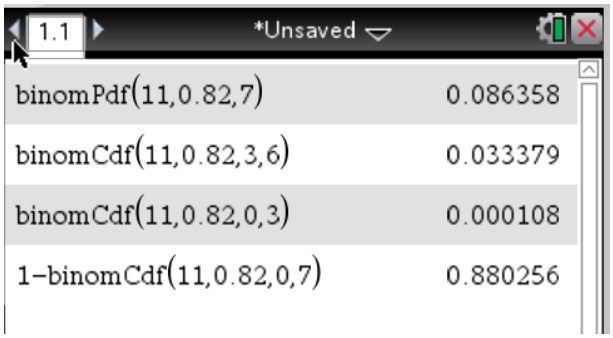
7

TI-Nspire CAS



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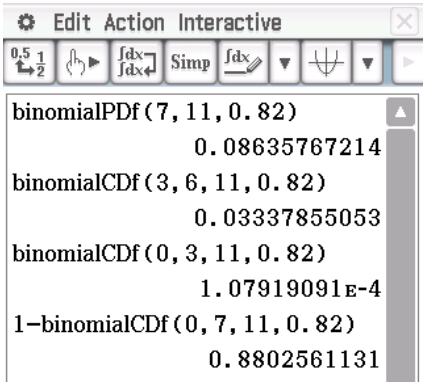


$n = 11$, $p = 0.82$

a $P(X = 7) = 0.0864$

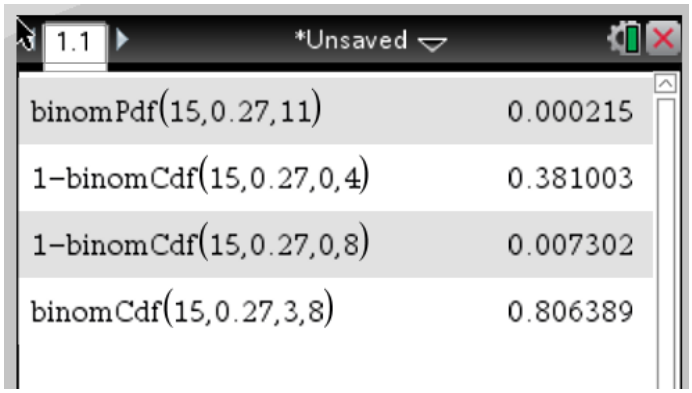
b $P(3 \leq X \leq 6) = P(X \leq 6) - P(X \leq 2) = 0.033\,386 - 0.000\,008 = 0.033\,38$

c $P(X \leq 3) = 0.000\,1079$

d $P(X > 7) = 1 - P(X \leq 7) = 1 - 0.119\,744 = 0.8803$

8

TI-Nspire CAS



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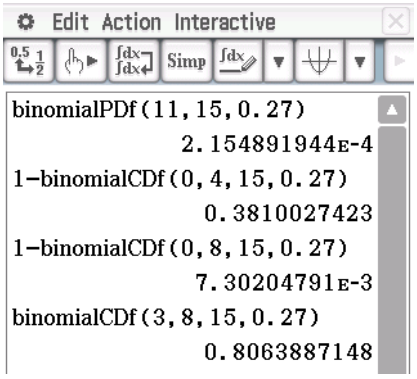


$n = 15$, $p = 0.27$

a $P(X = 11) = 0.0002$

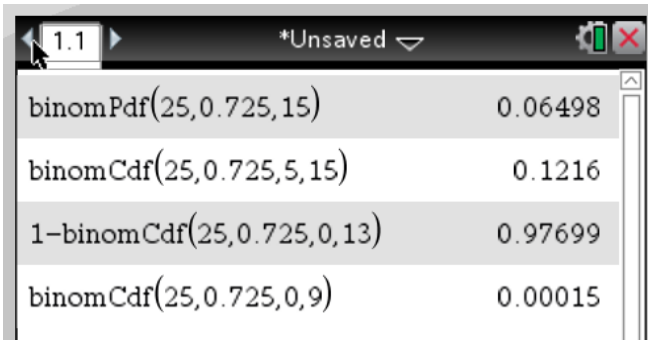
b $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.618\,998 = 0.381$

c $P(X \geq 9) = 1 - P(X \leq 8) = 0.0073$

d $P(2 < X < 9) = P(X \leq 8) - P(X \leq 2) = 0.9927 - 0.1863 = 0.8064$

9

TI-Nspire CAS



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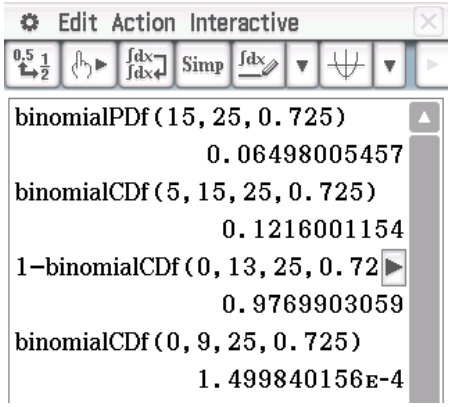

$n = 25$, $p = 0.725$

a $P(X = 15) = 0.0650$

b $P(5 \leq X \leq 15) = P(X \leq 15) - P(X \leq 4) = 0.1216$

c $P(X \geq 14) = 1 - P(X \leq 13) = 0.9770$

d $P(X < 10) = P(X \leq 9) = 0.0001$

10 $n = 8$, $p = 0.4$

a $P(2 \text{ successes occur}) = 0.2090$

b $P(5 \text{ successes occur}) = 0.1239$

c $P(\text{at least 2 successes occur}) = P(X \geq 2) = 1 - P(X \leq 1) = 0.8936$

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11

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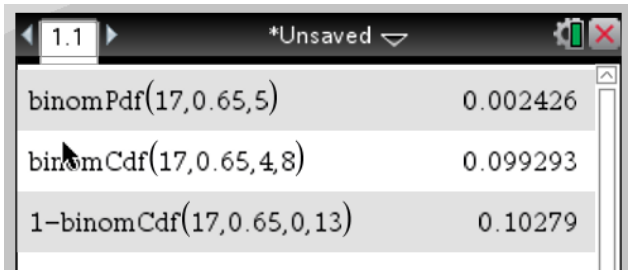

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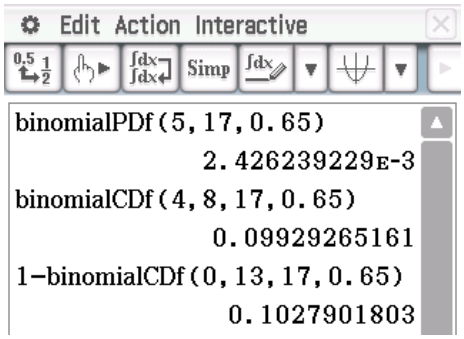


$n = 17$, $p = 0.65$

a $P(X = 5) = 0.0024$

b $P(4 \leq X \leq 8) = P(X \leq 8) - P(X \leq 3) = 0.0993$

c $P(X \geq 14) = 1 - P(X \leq 13) = 0.1028$

Reasoning and communication

12 $n = 6, p = 0.5$

a $P(X = x) = 0.3125$

$= {}^{6}C_{x}(0.5)^{x}(0.5)^{6-x}$

$x = 3$

b $P(X = x) = 0.093\,75$

$x = 1, 5$

c $P(X = x) = 0.234\,375$

$x = 2, 4$

13 $n = 5, q = 0.5$

a $P(X < x) = 0.1875$

$x = 2$ as $P(X \leq 1) = 0.1875$

b $P(X \geq x) = 0.1875$

$P(X \leq x - 1) = 0.8125$

$x - 1 = 3$, so $x = 4$

c $P(X > x) = 0.5$

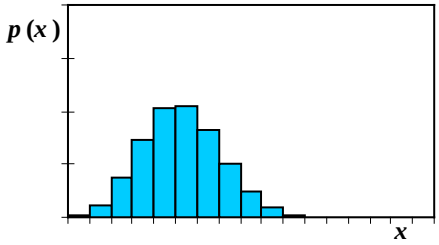
$P(X \leq x) = 0.5$

$x = 2$

Exercise 5.05 Properties of the binomial distribution

Concepts and techniques

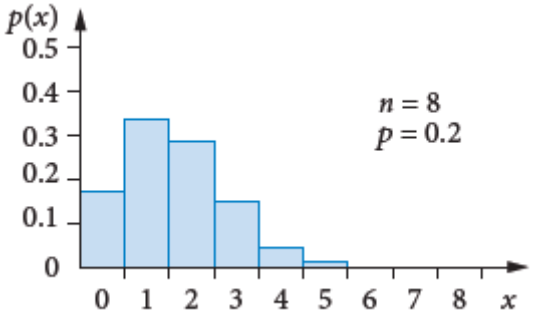
1 C as clustered around 0.3



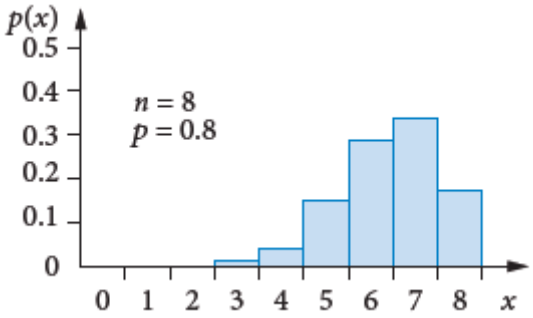
2 D $n = 32$ and $p = \frac{1}{4}$

$\mu = np = 8$, $\sigma^2 = npq = 6$

3 **a** $n = 8$ and $p = 0.2$

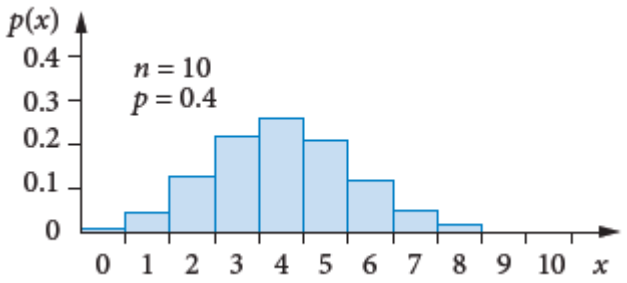


$n = 8$ and $p = 0.8$

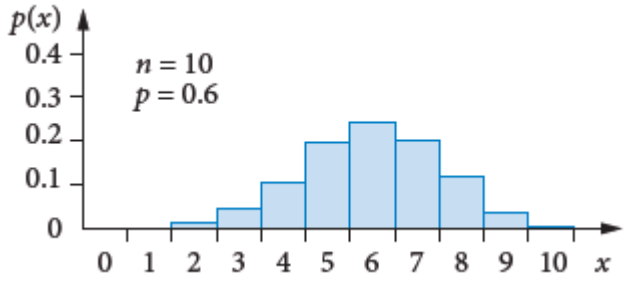


b The graphs are mirror images of each other, skewed positively and negatively respectively.

4 **a** $n = 10$ and $p = 0.4$

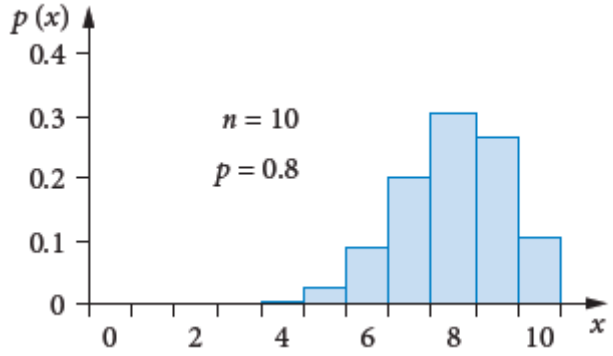


$n = 10$ and $p = 0.6$



b The graphs are mirror images of each other, skewed positively and negatively respectively.

5 **a**



b 8

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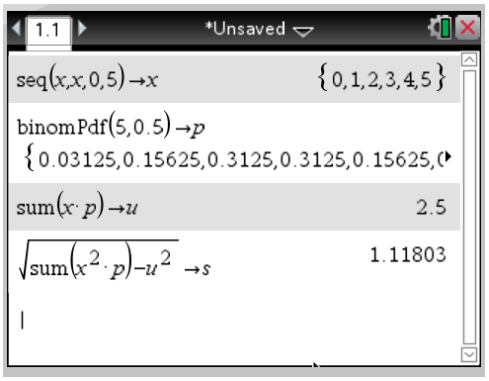

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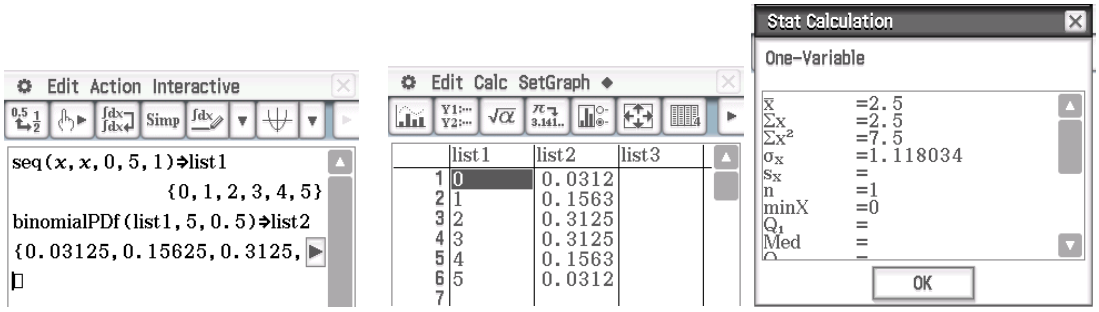

a $n = 5$ and $p = 0.5$

x	0	1	2	3	4	5	**Sums**
$P(X = x)$	0.0313	0.1563	0.3125	0.3125	0.1563	0.0313	1.0002
$x \times P(X = x)$	0	0.1563	0.625	0.9375	0.6252	0.1565	2.5005
$x^2 \times P(X = x)$	0	0.1563	1.25	2.8125	2.5008	0.7825	7.5021

$$\mu = E(X) = 0 \times 0.0313 + 1 \times 0.1563 + 2 \times 0.3125 + 3 \times 0.3125 + 4 \times 0.1563 + 5 \times 0.0313 = 2.5005$$

$$E(X^2) = 0^2 \times 0.0313 + 1^2 \times 0.1563 + 2^2 \times 0.3125 + 3^2 \times 0.3125 + 4^2 \times 0.1563 + 5^2 \times 0.0313 = 7.5021$$

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$\sigma_x^2 = E(X^2) - \mu^2$

$\sigma_x^2 = 7.5021 - 2.5005^2$

$= 1.2496$

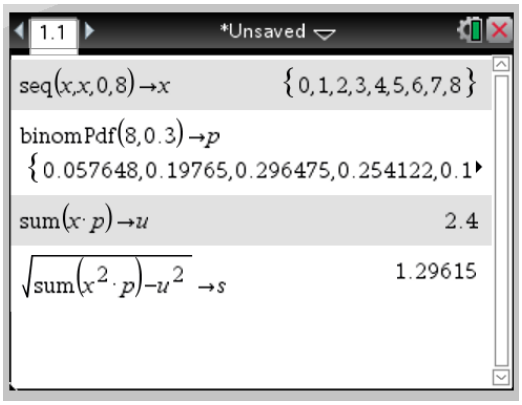
$\sigma_x \approx 1.118$

$\mu = np = 5 \times 0.5 = 2.5$

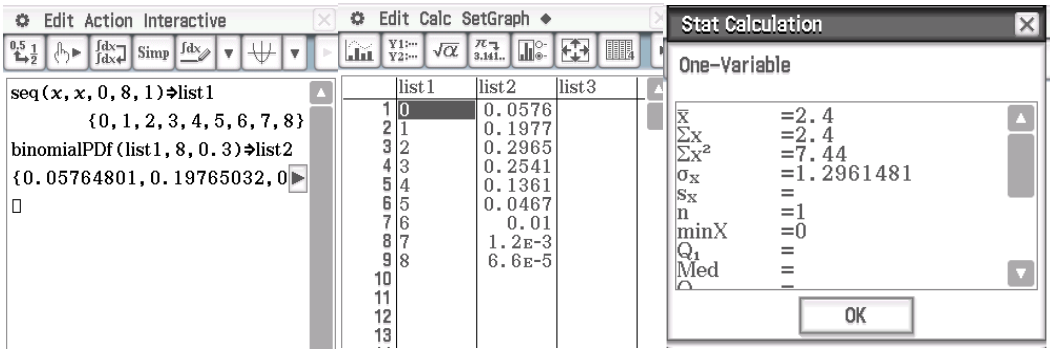
$\sigma = \sqrt{npq} = \sqrt{5 \times 0.5 \times 0.5} = 1.118$

b

TI-Nspire CAS



ClassPad



$n = 8$ and $p = 0.3$

x	0	1	2	3	4	5	6	7	8	**Sums**
$P(X = x)$	0.0576	0.1977	0.2965	0.2541	0.1361	0.0467	0.01	0.0012	0.0001	1
$x \times P(X = x)$	0	0.1977	0.593	0.7623	0.5444	0.2335	0.06	0.0084	0.0005	2.3998
$x^2 \times P(X = x)$	0	0.1977	1.186	2.2869	2.1776	1.1675	0.36	0.0588	0.0025	7.437

$\mu = E(X) = 2.3998$

$E(X^2) = 7.437$

$\sigma_x^2 = E(X^2) - \mu^2$

$\sigma_x^2 = 7.437 - 2.3998^2$

$= 1.68$

$\sigma_x \approx 1.296$

$\mu = np = 8 \times 0.3 = 2.4$

$\sigma = \sqrt{npq} = \sqrt{8 \times 0.3 \times 0.7} = 1.296$

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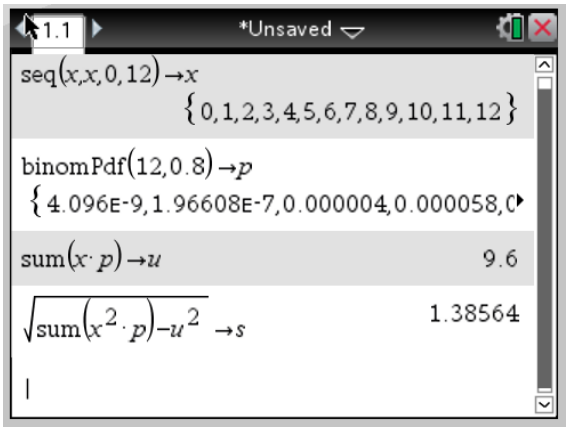
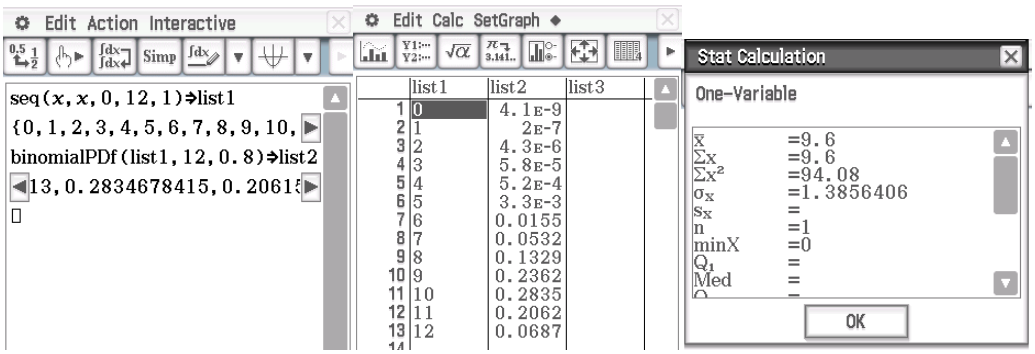
c

TI-Nspire CAS



ClassPad



$n = 12$ and $p = 0.8$

x	0	1	2	3	4	5	6
$P(X = x)$	0	0	0	0	0	0.0033	0.0155
$x \times P(X = x)$	0	0	0	0	0	0.0165	0.093
$x^2 \times P(X = x)$	0	0	0	0	0	0.0825	0.558

7	8	9	10	11	12	**Sums**
0.0532	0.1329	0.2362	0.2835	0.2062	0.0687	0.9995
0.3724	1.0632	2.1258	2.835	2.2682	0.8244	9.5985
2.6068	8.5056	19.1322	28.35	24.9502	9.8928	94.0781

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$\mu = E(X) = 9.5985$

$E(X^2) = 94.0781$

$\sigma_x^2 = E(X^2) - \mu^2$

$\sigma_x^2 = 94.0781 - 9.5985^2$

$= 1.9469$

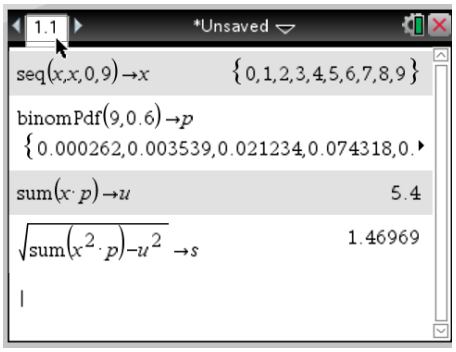
$\sigma_x \approx 1.39$

$\mu = np = 12 \times 0.8 = 9.6$

$\sigma = \sqrt{npq} = \sqrt{12 \times 0.8 \times 0.2} = 1.39$

d

TI-Nspire CAS



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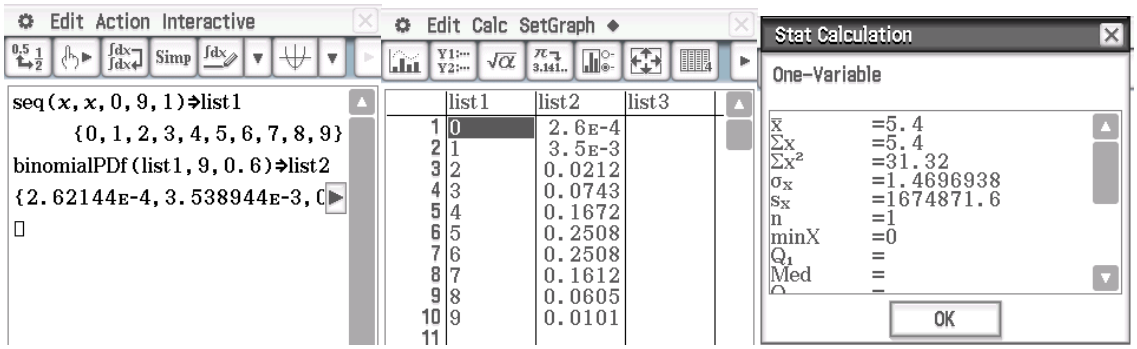

$n = 9$ and $p = 0.6$

x	0	1	2	3	4	5	6	7	8	9	**Sums**
$P(X = x)$	0.0003	0.0035	0.0212	0.0743	0.1672	0.2508	0.2508	0.1612	0.0605	0.0101	0.9999
$x \times P(X = x)$	0	0.0035	0.0424	0.2229	0.6688	1.254	1.5048	1.1284	0.484	0.0909	5.3997
$x^2 \times P(X = x)$	0	0.0035	0.0848	0.6687	2.6752	6.27	9.0288	7.8988	3.872	0.8181	31.3199

$\mu = E(X) = 5.4$

$E(X^2) = 31.32$

$\sigma_x^2 = E(X^2) - \mu^2$

$\sigma_x^2 = 31.32 - 5.4^2$

$= 2.16$

$\sigma_x \approx 1.47$

$\mu = np = 9 \times 0.6 = 5.4$

$\sigma = \sqrt{npq} = \sqrt{9 \times 0.6 \times 0.4} = 1.47$

7 **a** $n = 7$ and $p = 0.1$

$\mu = np = 7 \times 0.1 = 0.7$

$\sigma = \sqrt{npq} = \sqrt{7 \times 0.1 \times 0.9} = 0.794$

b $n = 7$ and $p = 0.9$

$\mu = np = 7 \times 0.9 = 6.3$

$\sigma = \sqrt{npq} = \sqrt{7 \times 0.9 \times 0.1} = 0.794$

c $n = 20$ and $p = 0.65$

$\mu = np = 20 \times 0.65 = 13$

$\sigma = \sqrt{npq} = \sqrt{20 \times 0.65 \times 0.35} = 2.133$

d $n = 30$ and $p = 0.34$

$\mu = np = 30 \times 0.34 = 10.2$

$\sigma = \sqrt{npq} = \sqrt{30 \times 0.34 \times 0.66} = 2.59$

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Reasoning and communication

8 $n = 10$, $Var(X) = 0.9 = npq$

$0.9 = 10p(1 - p)$

$p = 0.1$ or $p = 0.9$

9 A $\mu = np = 24$, $SD(X) = 3 = \sqrt{npq}$

$3 = \sqrt{24(1-p)}$

$9 = 24(1 - p)$

$p = 0.625 = \frac{5}{8}$

10 **a** Binomial as you either have an ace or NOT an ace.

b $\mu = np = 60 \times \frac{4}{52} = \frac{60}{13} = 4.62$

Expect about 5.

c About 95%.

11 $\mu = 5$, $\sigma = 2$

$\mu = np = 5$

$2 = \sqrt{npq}$

$4 = 5q$

$q = 0.8,\ p = 0.2,\ n = \frac{5}{0.2} = 25$

$P(X = 5) = 0.196$

12 $\mu = np = 200 \times 0.5 = 100$

13 $p = 0.8$, $\mu = 500$, $n = ?$

$\mu = np$

$500 = n \times 0.8$

$n = 625$

14 **a** $n = 420$ batteries, $\mu = np = 420 \times 0.4 = 168$

b $\sigma = \sqrt{npq} = \sqrt{420 \times 0.4 \times 0.6} = 10.04$

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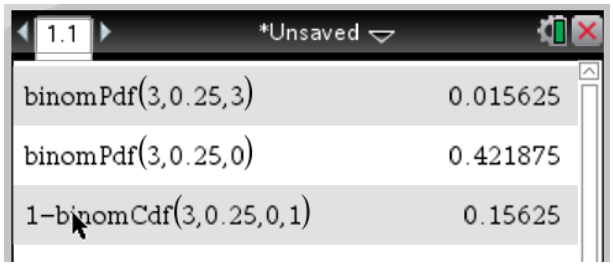
Exercise 5.06 Applications of the binomial distribution

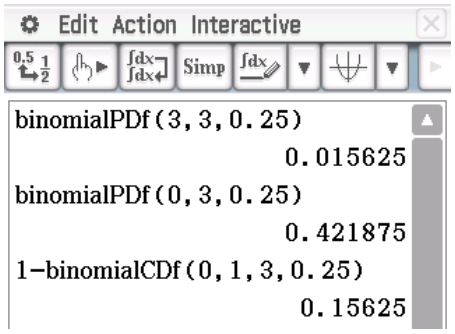
Concepts and techniques

1

TI-Nspire CAS



ClassPad



a $n = 3$, $p = 0.25$ and $q = 0.75$

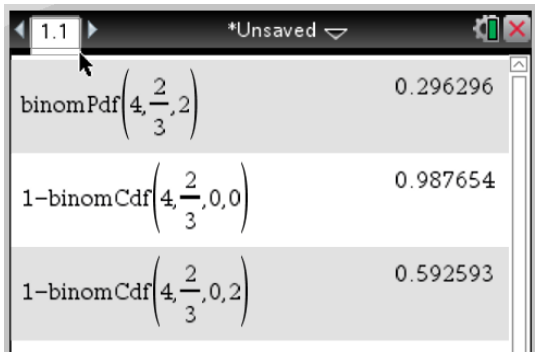
b $P(x = 3) = 0.0156\ 25$

c $P(x = 0) = 0.421\ 875$

d $P(\text{at least 2 flowers are red}) = 1 - P(x \leq 1) = 0.156\ 25$

2

TI-Nspire CAS



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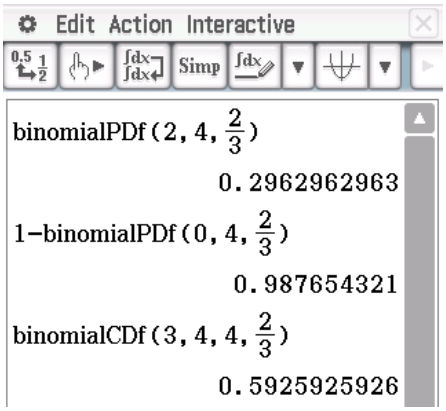


$n = 4$, $p = \frac{2}{3}$

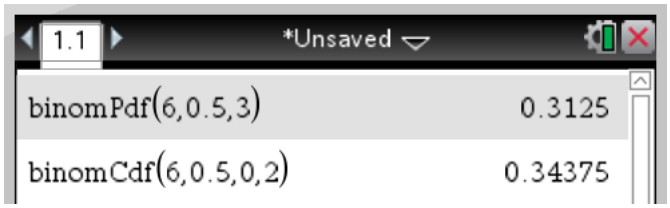
a $P(x = 2) = 0.296$

b $P(x \geq 1) = 1 - P(x = 0) = 0.988$

c $P(x > 2) = 1 - P(x \leq 2) = 0.593$

3

TI-Nspire CAS



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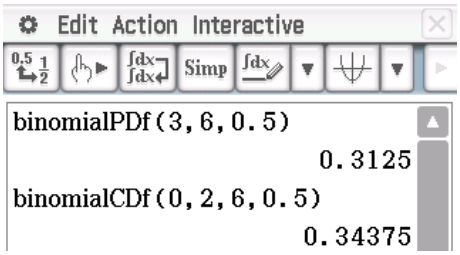


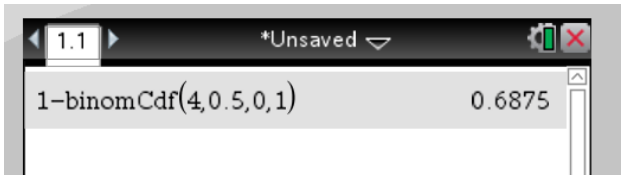
$n = 6$ children, $p = 0.5$

a $P(3 \text{ boys and } 3 \text{ girls}) = 0.3125$

b $P(x = 0 \text{ or } 1 \text{ or } 2) = P(x \leq 2) = 0.34375$

4

TI-Nspire CAS



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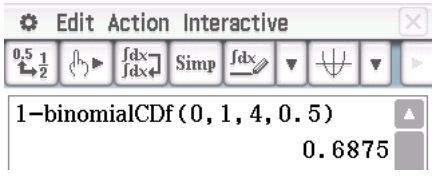

$n = 4$ children, $p = 0.5$

$P(x \geq 2) = 1 - P(x \leq 1) = 0.6875$

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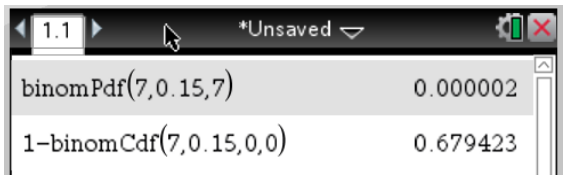
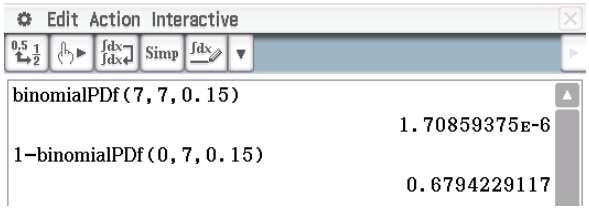
5 **TI-Nspire CAS**



ClassPad

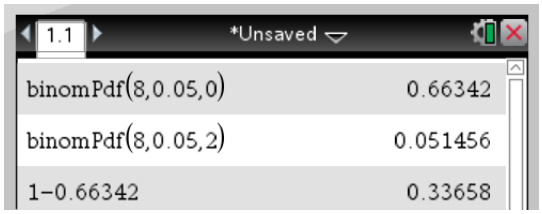


$n = 7$, $p = 0.15$

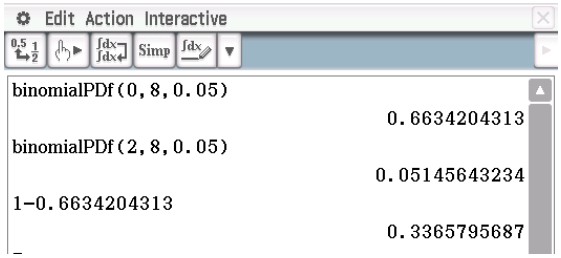
a $P(x = 7) = 0.000\ 001\ 7$

b $P(x \geq 1) = 1 - P(x = 0) = 0.679$

6 **TI-Nspire CAS**



ClassPad



$n = 8$, $p = 0.05$

a $P(x = 0) = 0.663$

b $P(x = 2) = 0.051$

c $P(x \geq 1) = 1 - P(x = 0) = 0.337$

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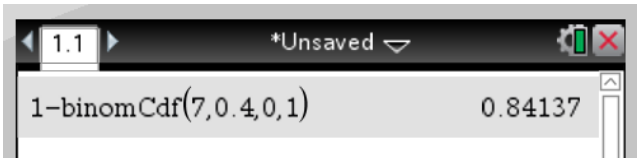
7 **a** $(0.4)^2(0.6)^5 = 0.012$

b $(0.4)^2 = 0.16$

c

TI-Nspire CAS



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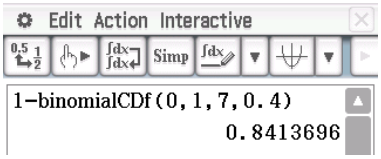
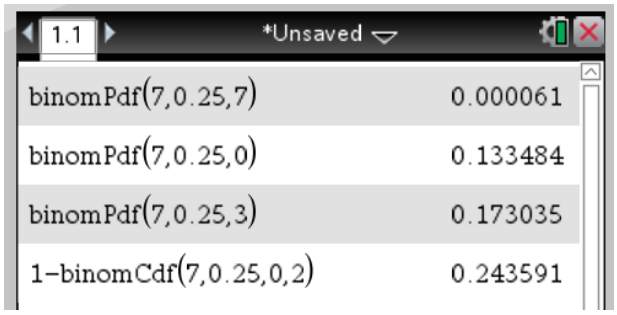


$n = 7$, $p = 0.4$, $P(x \geq 2) = 1 - P(x \leq 1) = 0.84$

8

TI-Nspire CAS



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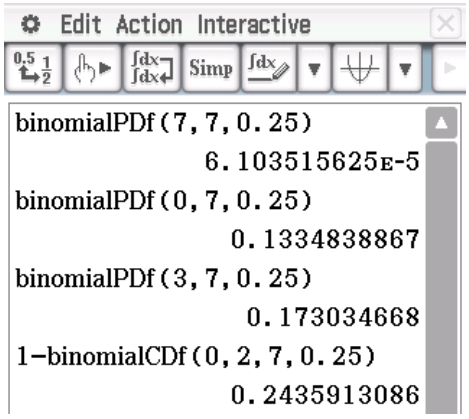


$n = 7$, $p = 0.25$

a $P(x = 7) = 0.000\,062$

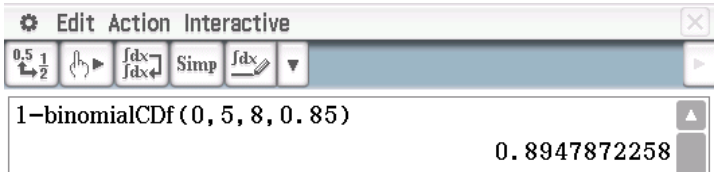
b $P(x = 0) = 0.133$

c $P(x = 3) = 0.173$

d $P(x \geq 3) = 1 - P(x \leq 2) = 0.243$

9 **TI-Nspire CAS**

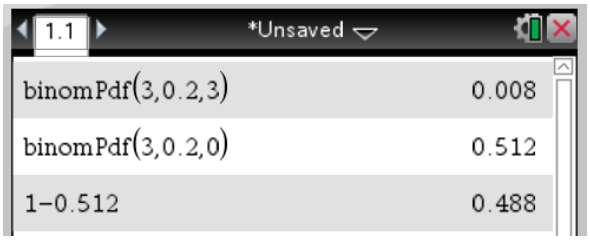
ClassPad



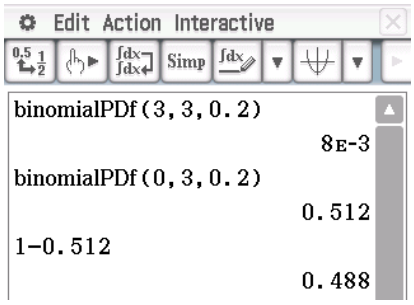
$n = 8$, $p = 0.85$

$P(x \geq 6) = 1 - P(x \leq 5) = 0.895$

10 **TI-Nspire CAS**



ClassPad



a $n = 3$, $p = 0.2$, $q = 0.8$

b $P(x = 3) = 0.008$

c $P(x = 0) = 0.512$

d $P(x \geq 1) = 1 - P(x = 0) = 0.488$

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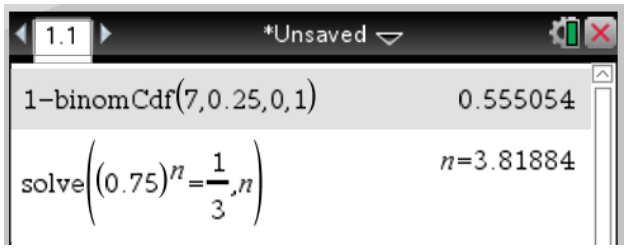
Reasoning and communication

11

TI-Nspire CAS



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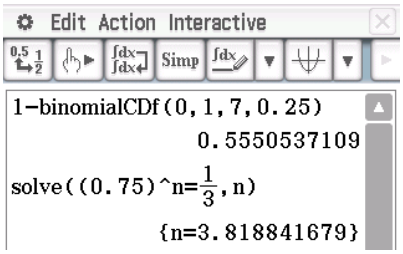


$p = \frac{1}{4}$

a $n = 7$, $P(X \geq 2) = 1 - P(X \leq 1) = 0.555$

b $n = ?$, $P(X \geq 1) > \frac{2}{3}$

$P(X \geq 1) = 1 - P(x = 0) > \frac{2}{3}$

i.e. $P(x = 0) < \frac{1}{3}$

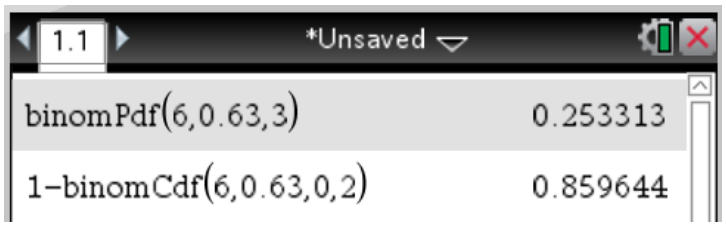
$\Rightarrow \binom{n}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^n < \frac{1}{3}$

$\Rightarrow \left(\frac{3}{4}\right)^n < \frac{1}{3}$

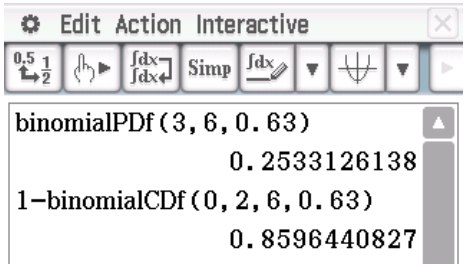
$n = 4$

12

TI-Nspire CAS



ClassPad



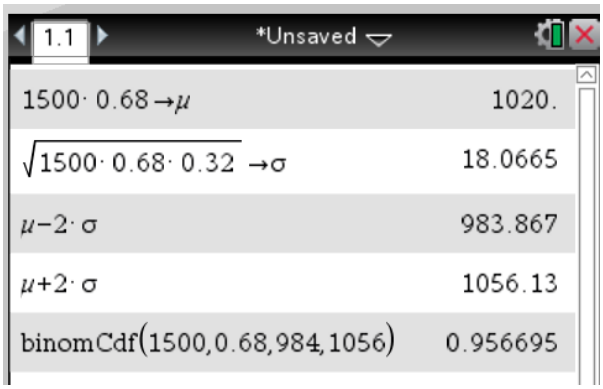
$n = 6$, $p = 0.63$

a $P(x = 3) = 0.253$

b $P(x \geq 3) = 1 - P(x \leq 2) = 0.860$

13

TI-Nspire CAS



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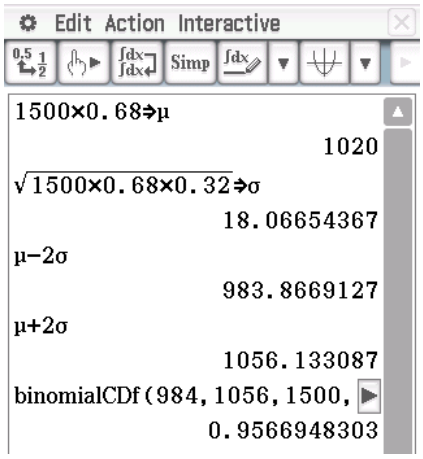

$n = 1500$, $p = 0.68$

$\mu = np = 1500 \times 0.68 = 1020$

$\sigma = \sqrt{npq} = \sqrt{1500 \times 0.68 \times 0.32} = 18.066...$

$2\sigma \approx 36.133...$

$\mu + 2\sigma = 1056$, $\mu - 2\sigma = 984$

There is a 95% chance that between 984 and 1056 students will attend a government school.

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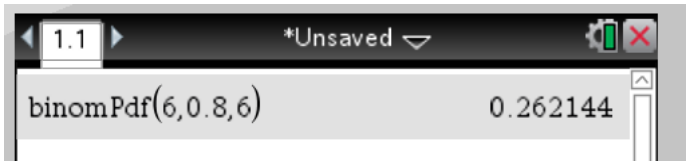
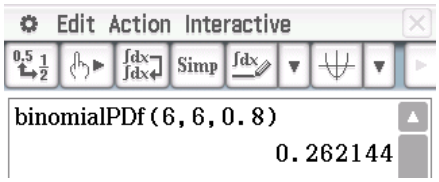
14

TI-Nspire CAS



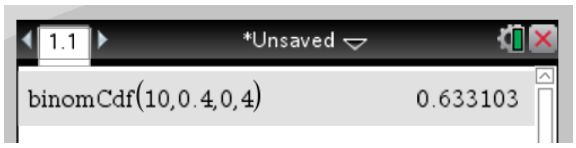
ClassPad



$n = 6$, $p = 0.8$, $P(x = 6) = 0.262$

15

TI-Nspire CAS



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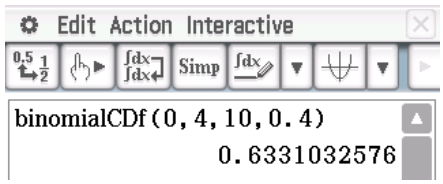
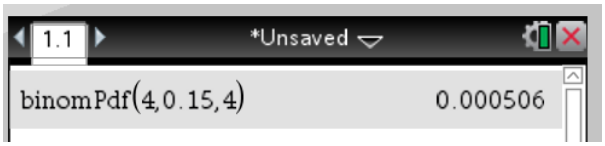


$n = 10$, $p = 0.4$, $P(x < 5) = P(x \leq 4) = 0.633$

16

TI-Nspire CAS



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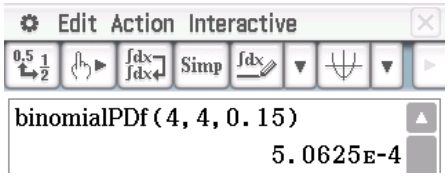
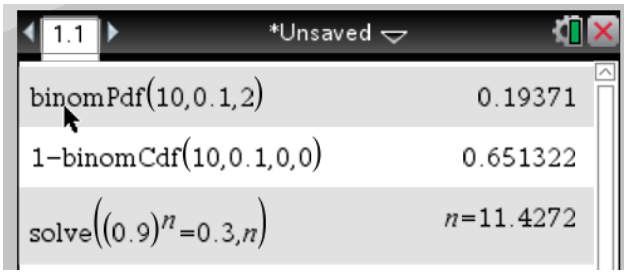
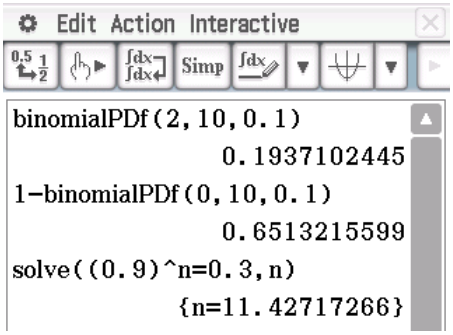


$n = 4$, $p = 0.15$, $P(x = 4) = 0.0005$

17 TI-Nspire CAS



ClassPad



$p = 0.1$

a $n = 10$

i $P(x = 2) = 0.194$

ii $P(x \geq 1) = 1 - P(x = 0) = 0.651$

b $n = ?$

$P(x \geq 1) > 0.7$

$1 - P(x = 0) > 0.7$

$P(x = 0) < 0.3$

${}^nC_0(0.1)^0(0.9)^n < 0.3$

i.e. $(0.9)^n < 0.3$

$n \log_e(0.9) < \log_e(0.3)$

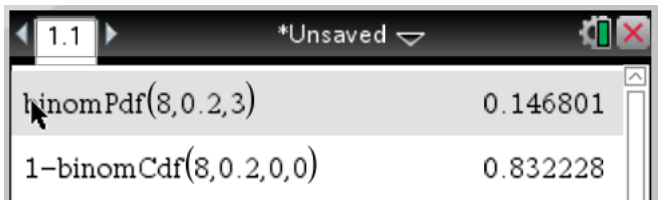
[Note: $\log_e(0.9) < 0$]

$$n > \frac{\log_e(0.3)}{\log_e(0.9)} = 11.42$$

$n = 12$

18

TI-Nspire CAS



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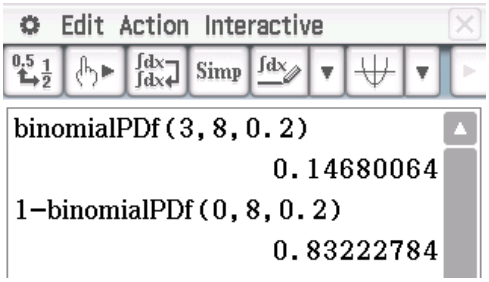


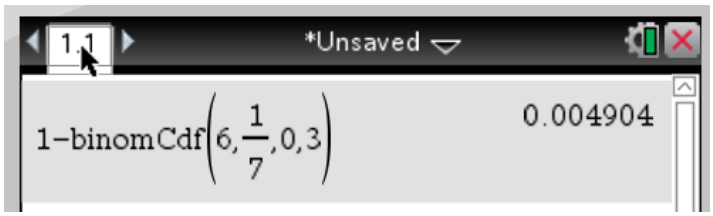
$p = 0.2$, $n = 8$

a $P(x = 3) = 0.147$

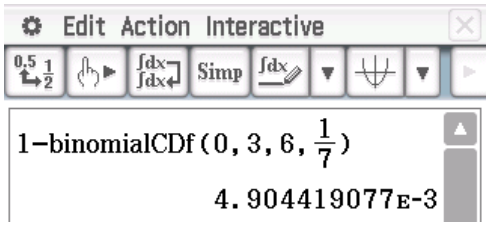
b $P(x \geq 1) = 1 - P(x = 0) = 0.832$

19

TI-Nspire CAS



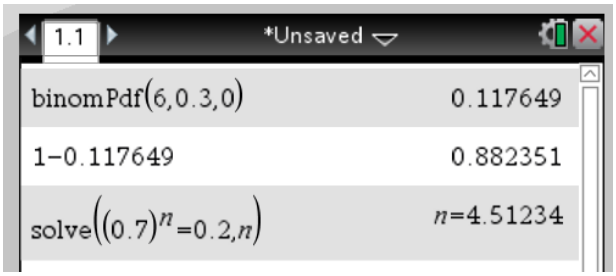
ClassPad



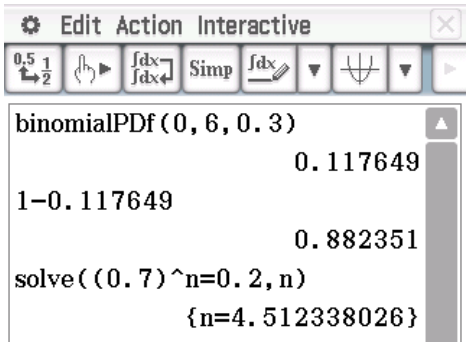
$p = \frac{1}{7}$, $n = 6$

$P(x \geq 4) = 1 - P(x \leq 3) = \frac{577}{117\ 649}$

20 TI-Nspire CAS



ClassPad



$p = 0.3$

a $n = 6$

i $P(x = 0) = 0.1176$

ii $P(x \geq 1) = 1 - P(x = 0) = 0.882$

b $n = ?$

$P(x \geq 1) > 0.8$

$1 - P(x = 0) > 0.8$

$P(x = 0) < 0.2$

${}^nC_0(0.7)^n(0.3)^0 < 0.2$

i.e. $(0.7)^n < 0.2$

$n \log_e(0.7) < \log_e(0.2)$

[Note: $\log_e(0.7) < 0$]

$$n > \frac{\log_e(0.2)}{\log_e(0.7)} = 4.51$$

$n = 5$

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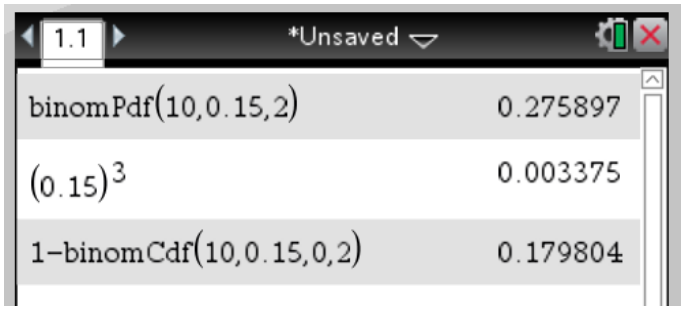
21

TI-Nspire CAS



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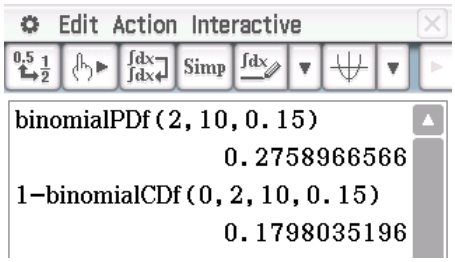


$n = 10$, $p = 0.15$

a $P(x = 2) = 0.276$

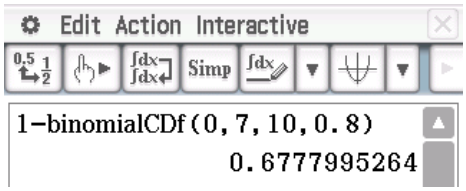
b This is not binomial, $P(\text{first 3 defective}) = (0.15)^3 = 0.0033\ 75$

c $P(x \geq 3) = 1 - P(x \leq 2) = 0.18$

22

TI-Nspire CAS

ClassPad



$n = 10$, $p = 0.8$

$P(x \geq 8) = 1 - P(x \leq 7) = 0.678$

Chapter 5 Review

Multiple choice

1 B As there is either a six or not a six. Two possible outcomes.

2 C As $\sigma = \sqrt{pq} = \sqrt{0.2 \times 0.8} = \sqrt{0.16} = 0.4$.

3 D 'the number of cards drawn from a well shuffled deck of playing cards before an ace occurs.' as the sequence is $q, q, q, ...,p$

4 E $P(X = 3) = (0.6)^3(0.4) = 0.0864$

5 A 'the number of hearts that occur when 7 cards are drawn with replacement from a well-shuffled deck of 52 cards' as number of hearts are successes and not hearts are failures.

6 E The binomial distribution with $n = 12$ and $p = 0.25$.

7 B $n = 7, p = \frac{5}{12}, P(x \leq 1) = 0.1379$

8 D $n = 20, p = 0.9, P(x \geq 18) = 1 - P(x \leq 17) = 0.677$

9 D $\mu = np = 20 \times 0.7 = 14$

Short answer

10 **a** No as there is more than one suit. Binomial needs success and failure – two options.

b No, haven't got two alternatives only.

c Yes, as orange or not orange only.

d No, there are up to 15 possible tails.

e Yes, as a king or not a king.

f No, as there are up to 3 blue marbles.

g Yes, as either left-handed or not.

11 $p = \frac{8}{45}$

12 $p = \frac{1}{6}, \sigma^2 = npq = 1 \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$

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13 $p = \frac{1}{3}$

$P(x = 5) = \left(\frac{2}{3}\right)^4 \times \frac{1}{3} = \frac{16}{243}$

14 $p = 0.3$

a $P(X = 3) = (0.7)^3(0.3) = 0.1029$

b $P(X \leq 5) = (0.3) + (0.7)^1(0.3) + (0.7)^2(0.3) + (0.7)^3(0.3) + (0.7)^4(0.3) + (0.7)^5(0.3)$

$= 0.3 + 0.3 \times 0.7(1 + 0.7 + 0.7^2 + ... + 0.7^4)$

$= 0.8824$

c Assuming she keeps going round and round until she gets a win,

$E(X) = \frac{1-p}{p} = \frac{1-0.3}{0.3} = 2.333\ldots$

15 $p = 0.68$

a $P(Y = 3) = (0.32)^3(0.68) = 0.0223$

b $P(2 \leq Y \leq 9) = (0.32)^2 (0.68) + (0.32)^3(0.68) + (0.32)^4 (0.68) + (0.32)^5(0.68)$

$+ (0.32)^6(0.68) + (0.32)^7(0.68) + (0.32)^8(0.68) + (0.32)^9(0.68)$

$= (0.32)^2 (0.68)[1 + (0.32) + (0.32)^2 + (0.32)^3 + (0.32)^4 + (0.32)^5 + (0.32)^6 + (0.32)^7]$

$= 0.1024$

16 **a** Yes, there are two alternatives, prime or not prime.

b No, the outcomes vary.

c Yes, binomial. The card is either a spade or it is not.

d No, not binomial. The number of tosses required can vary.

e Yes, binomial. The die shows either an even number or not an even number.

f Without replacement means the probability of obtaining green changes, so not binomial.

g Recording heads may vary. Not binomial.

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17 **a** $\binom{5}{2}(0.4)^2(0.6)^3 = 0.3456$

b $\binom{10}{7}(0.43)^7(0.57)^3 = 0.604$

c $\binom{8}{2}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^6 .= 0.001\ 71$

18 **a** $n = 8$

b $p = 0.45$

c

x	0	1	2	3	4	5	6	7	8
$P(X = x)$	0.0084	0.0548	0.1569	0.2568	0.2627	0.1719	0.0703	0.0164	0.0017

19 $n = 8, p = 0.35$

a $P(X = 7) = 0.0033$

b $P(x \geq 2) = 1 - P(x \leq 1) = 0.831$

c $P(X > 4) = 1 - P(x \leq 4) = 0.106$

20 $n = 5, p = \frac{12}{52}$

$P(X = 2) = 0.242$

21 $n = 20, p = 0.68$

a $P(X = 12) = 0.1354$

b $P(X \geq 7) = 1 - P(x \leq 6) = 0.9994$

c $P(X \leq 9) = 0.0279$

22 $n = 9, p = 0.6$

a $P(x = 3) = 0.0743$

b $P(x = 6) = 0.2508$

c $P(X \geq 2) = 1 - P(x \leq 1) = 0.9962$

23 $n = 3, p = 0.7$

$P(X \geq 2) = 1 - P(x \leq 1) = 0.784$

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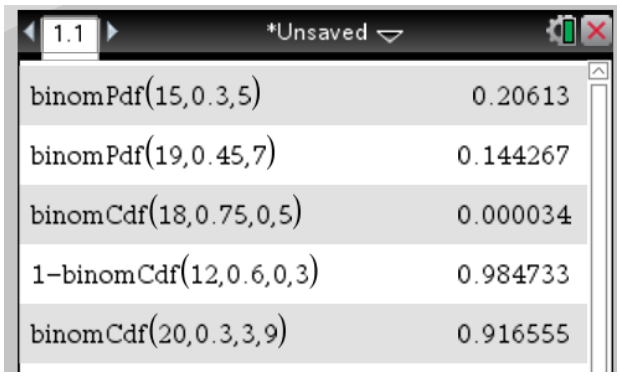
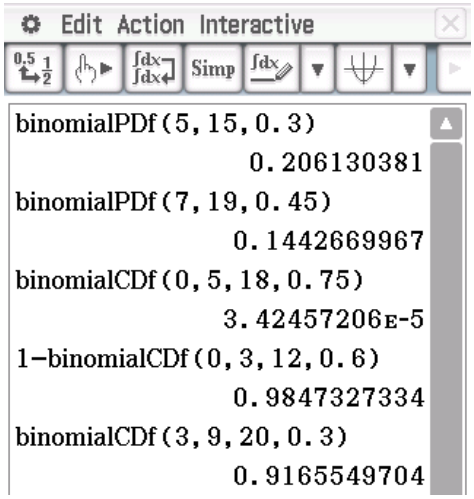
24 **TI-Nspire CAS**



ClassPad



a $n = 15$ and $p = 0.3$

$P(x = 5) = 0.2061$

b $n = 19$ and $p = 0.45$

$P(x = 7) = 0.1442$

c $n = 18$ and $p = 0.75$

$P(x < 6) = P(x \leq 5) = 0.000\,034\ldots$

d $n = 12$ and $p = 0.6$

$P(x \geq 4) = 1 - P(x \leq 3) = 0.9847$

e $n = 20$ and $p = 0.3$

$P(3 \leq x \leq 9) = P(x \leq 9) - P(x \leq 2) = 0.9166$

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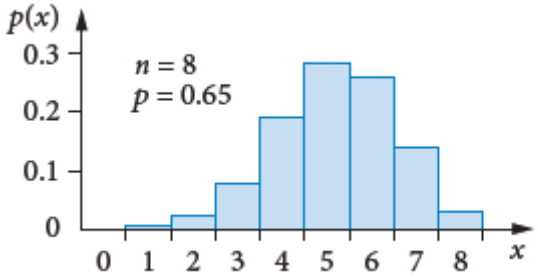
25 $n = 12$ and $p = 0.5$

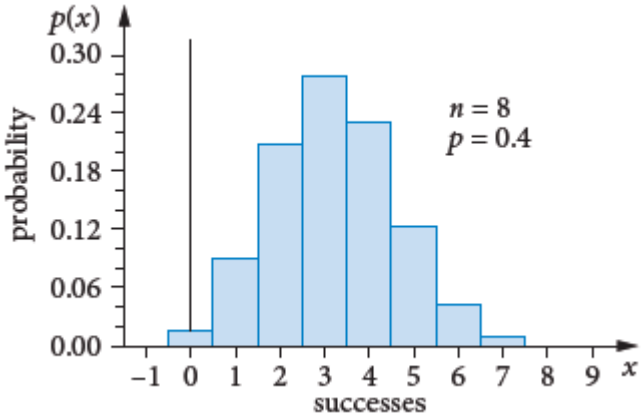
$P(3 < x < 7) = P(x \leq 6) - P(x \leq 3) = 0.5398$

26 $n = 8$ and $p = 0.65$



Slightly skewed to the right. i.e., slightly negatively skewed.

27 **a** $n = 8$ and $p = 0.4$



b The most likely number of successes is 3.

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28 **a** $n = 8$ and $p = 0.2$

$\mu = np = 8 \times 0.2 = 1.6$

$\sigma = \sqrt{npq} = \sqrt{8 \times 0.2 \times 0.8} = \sqrt{1.28} = 1.131$

b $n = 10$ and $p = 0.7$

$\mu = np = 10 \times 0.7 = 7$

$\sigma = \sqrt{npq} = \sqrt{10 \times 0.7 \times 0.3} = \sqrt{2.1} = 1.45$

c $n = 15$ and $p = 0.55$

$\mu = np = 15 \times 0.55 = 8.25$

$\sigma = \sqrt{npq} = \sqrt{15 \times 0.55 \times 0.45} = \sqrt{3.7125} = 1.93$

29 $n = 20$ and $p = 0.15$

a $P(x = 7) = 0.016$

b $P(X \geq 7) = 1 - P(x \leq 6) = 0.0219$

c $P(X \geq 5) = 1 - P(x \leq 4) = 0.1702$

30 $n = 12$ and $p = 0.28$

a $(0.28)^2 \times (0.72)^{10} = 0.002\,935$

b $P(X = 2) = 0.1937$

c $P(X \geq 2) = 1 - P(x \leq 1) = 0.8900$

Application

31 $n = 10$ and $p = 0.\ 8$

a $P(X = 10) = 0.1074$

b $P(X = 9) = 0.2684$

c $P(X \geq 7) = 1 - P(x \leq 6) = 0.8791$

32 $n = 15$ and $p = 0.\ 85$

a $P(X = 15) = (0.85)^{15} = 0.087\,354$

b Use $q = 0.15$ as success

P(at least 2 have no immunity to the disease) = $P(X \geq 2) = 1 - P(x \leq 1) = 0.6814$

c Use $q = 0.15$ as success

$P(X < 4) = P(X \leq 3) = 0.8227$

33 $n = 15$ and $p = 0.05$ (defective)

a $P(\text{all are operative}) = P(X = 0) = 0.4633$

b $P(X = 2) = 0.1348$

c $P(X \geq 2) = 1 - P(x \leq 1) = 0.171$

d $P(X \leq 2) = 0.9638$

34 $n = 18$ and $p = 0.3$

$P(X \leq 4) = 0.3327$

35 $n = 25$ and $p = 0.18$

a $\mu = np = 25 \times 0.18 = 4.5$

4 or 5 are expected to be iron deficient.

b $\sigma = \sqrt{npq} = \sqrt{25 \times 0.18 \times 0.82} = 1.920\,94$

$\mu + 2\sigma = 4.5 + 2(1.92) = 8.342$

$\mu + 2\sigma = 4.5 - 2(1.92) = 0.658$

$P(0.658 < x < 8.342)$

$= P(0 < x \leq 8)$

$= P(X \leq 8) - P(X = 0)$

$= 0.9678$

c There is about a 97% chance that from 1 to 8 women in the study group will suffer from iron deficiency.

36 $\mu = 12 = np$, $\sigma = 3 = \sqrt{npq}$

$9 = 12q$

$q = 0.75$, $p = 0.25$

$12 = n(0.25)$

$n = 48$

$n = 48$, $p = 0.25$, $P(X = 9) = 0.085\,78$

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37 $p = 0.18$ (objects to fluoridisation)

a $n = 10$

i $P(X = 0) = 0.1374$

ii $P(X \geq 2) = 1 - P(x \leq 1) = 0.5608$

b $n = ?$

$P(x \geq 1) > 0.9$

$1 - P(x = 0) > 0.9$

$P(x = 0) < 0.1$

${}^nC_0(0.18)^0(0.82)^n < 0.1$

i.e. $(0.82)^n < 0.1$

$n \log_e (0.82) < \log_e (0.1)$

[Note: $\log_e (0.82) < 0$]

$$n > \frac{\log_e (0.1)}{\log_e (0.82)} = 11.6$$

$n = 12$