

NELSON SENIOR MATHS METHODS 12

FULLY WORKED SOLUTIONS

Chapter 5 Binomial distributions

Exercise 5.01 The Bernoulli distribution

Concepts and techniques

1 **b, c** and **e** each have two outcomes, one of which can be considered as a success and has a fixed probability

2 **E** $1 - \frac{3}{30} = \frac{27}{30} = \frac{9}{10}$

3 **E** $\frac{2}{5}$, $p = \frac{1}{5}$, $q = \frac{4}{5}$ and $\sigma = \sqrt{p(1-p)} = \sqrt{\frac{1}{5} \times \frac{4}{5}} = \frac{2}{5}$

4 **a** Yes, as there are two possible outcomes.

b No, as there are more than two possible outcomes

c No, as there are more than two possible outcomes.

d Yes, as there are two possible outcomes.

e Yes, as there are two possible outcomes.

f No, as there are more than two possible outcomes.

g No, as there are more than two possible outcomes.

5 **a** $P(\text{a red onion}) = 0.5$

b $P(\text{a six}) = \frac{1}{8}$

c $P(\text{a sherbet}) = \frac{1}{4}$

- 6**
- a** $p = \frac{2}{5}, q = \frac{3}{5}, n = 1$ Variance = $\sigma^2 = pq = \frac{6}{25}$
- b** $q = \frac{1}{3}, p = \frac{2}{3}, n = 1$ Variance = $\sigma^2 = pq = \frac{2}{9}$
- c** $p = 0.6, q = 0.4, n = 1$ Variance = $\sigma^2 = pq = 0.24$
- d** $q = 0.3, p = 0.7, n = 1$ Variance = $\sigma^2 = pq = 0.21$
- e** $p = \frac{3}{4}, q = \frac{1}{4}, n = 1$ Variance = $\sigma^2 = pq = \frac{3}{16}$
- 7**
- a** $q = 0.4, p = 0.6, n = 1$ Standard deviation = $\sigma = \sqrt{pq} = \sqrt{0.24} = 0.49$
- b** $p = \frac{2}{3}, q = \frac{1}{3}, n = 1$ Standard deviation = $\sigma = \sqrt{pq} = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$
- c** $q = \frac{24}{25}, p = \frac{1}{25}, n = 1$ Standard deviation = $\sigma = \sqrt{pq} = \sqrt{\frac{24}{625}} = \frac{2\sqrt{6}}{25}$
- d** $p = 0.2, q = 0.8, n = 1$ Standard deviation = $\sigma = \sqrt{pq} = \sqrt{0.16} = 0.4$
- e** $p = \frac{7}{9}, q = \frac{2}{9}, n = 1$ Standard deviation = $\sigma = \sqrt{pq} = \sqrt{\frac{14}{81}} = \frac{\sqrt{14}}{9}$

Reasoning and communication

8 $P(x=0) = \frac{23}{25} \times \frac{23}{25} \times \frac{23}{25} \times \frac{23}{25} \times \frac{23}{25} = \frac{6\,436\,343}{9\,765\,625} \approx 0.659\,082$

or $P(x=0) = \frac{{}^{92}C_5}{{}^{100}C_5} = 0.65908$

- 9** Two red, three green and one blue marble.

$$P(\text{a red marble}) = \frac{1}{3}.$$

$$\text{Standard deviation} = \sigma = \sqrt{pq} = \sqrt{\frac{1}{3} \times \frac{2}{3}} = \frac{\sqrt{2}}{3}$$

10 $P(\text{ace}) = \frac{4}{52} = \frac{1}{13}.$

$$p = \frac{1}{13}, q = \frac{12}{13} \quad \text{Variance} = \sigma^2 = pq = \frac{12}{169}$$

11 $P(x=0) = 0.8 \times 0.8 \times 0.8 \times 0.8 \times 0.8 = 0.8^5 = 0.32768$

12 $200 = 20 \text{ misshaped} + 180 \text{ normal}$

$$\begin{aligned} P(\text{at most } 1) &= P(x=0) + P(x=1) \\ &= \frac{{}^{20}C_0 \times {}^{180}C_{10}}{{}^{200}C_{10}} + \frac{{}^{20}C_1 \times {}^{180}C_9}{{}^{200}C_{10}} \\ &\approx \frac{1 \times 7.63 \times 10^{15}}{2.245 \times 10^{16}} + \frac{20 \times 4.4609 \times 10^{14}}{2.245 \times 10^{16}} \\ &= 0.33977 + 0.39740 \\ &\approx 0.73717 \end{aligned}$$

Exercise 5.02 The geometric distribution

Concepts and techniques

- 1** **D** ‘the number of digits I read beginning at a randomly selected point in a table of random digits until I find a 9’

This is failure, failure, ..., failure, success.

- 2** **B** because $(0.2)^4(0.8) = 0.00128$

- 3** **D** 0.0723 , $P(X = 4) = (0.7)^4(0.3) = 0.07203$

- 4** **B** $P(X > 3) = 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3)$
 $= 1 - [0.8 + 0.2 \times 0.8 + (0.2)^2 \times 0.8 + (0.2)^3 \times 0.8]$
 $= 0.0016$

- 5** **A** $E(X) = \frac{1-p}{p} = \frac{1-0.8}{0.8} = 0.25$

- 6** **a** Yes, as failure, failure, success with the same probabilities.

b No, as the probabilities do not stay the same.

c Yes, as a sequence of q, q, \dots, q, p with the same probabilities.

d Yes, as a sequence of B, B, ..., B, G with the same probabilities.

e Yes, as a sequence of N, N, ..., N, Y with the same probabilities.

- 7** $P(\text{exactly six rolls are required}) = \left(\frac{4}{6}\right)^5 \left(\frac{2}{6}\right) = 0.0439$

- 8** $P(H) + P(MH) + P(MMH) = \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{9+6+4}{27} = \frac{19}{27} = 0.704$

- 9** **a** $P(X = 5) = 0.8^5 \times 0.2 = 0.0655$

b $P(X \leq 5) = 0.2 + 0.8^1 \times 0.2 + 0.8^2 \times 0.2 + 0.8^3 \times 0.2 + 0.8^4 \times 0.2 + 0.8^5 \times 0.2$
 $= 0.2(1 + 0.8 + 0.8^2 + 0.8^3 + 0.8^4 + 0.8^5)$
 $= 0.738$

c $E(X) = \frac{1-p}{p} = \frac{1-0.2}{0.2} = 4$

10 $P(\text{underweight}) = \frac{3}{4}$

a $P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{1}{4} = \frac{3}{4}$

b
$$P(1 \leq X \leq 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$

$$= \frac{3}{4} \times \frac{1}{4} + \left(\frac{3}{4}\right)^2 \times \frac{1}{4} + \left(\frac{3}{4}\right)^3 \times \frac{1}{4} + \left(\frac{3}{4}\right)^4 \times \frac{1}{4} + \left(\frac{3}{4}\right)^5 \times \frac{1}{4}$$

$$= \frac{3}{4} \times \frac{1}{4} \left[1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \left(\frac{3}{4}\right)^4 \right] = 0.572$$

11 $P(\text{hit his drive straight}) = 0.1$

a
$$P(4 \leq X \leq 14) = (0.9)^4(0.1) + (0.9)^5(0.1) + \dots + (0.9)^{14}(0.1)$$

$$= (0.9)^4(0.1)\{1 + 0.9 + 0.9^2 + \dots + (0.9)^{10}\}$$

(Note: Geometric series)

$$= (0.9)^4(0.1)\{6.861\ 894\}$$

$$= 0.45$$

b
$$E(X) = \frac{1-p}{p} = \frac{1-0.1}{0.1} = 9$$

12 $P(\text{correct}) = 0.1$

Y denotes the number of questions answered for the first correct answer.

a $P(Y = 7) = (0.9)^6(0.1) = 0.0531$

b
$$P(2 \leq Y \leq 8) = (0.9)(0.1) + (0.9)^2(0.1) + (0.9)^3(0.1) + (0.9)^4(0.1) + (0.9)^5(0.1)$$

$$+ (0.9)^6(0.1) + (0.9)^7(0.1)$$

$$= (0.9)(0.1)\{1 + (0.9)^1 + (0.9)^2 + (0.9)^3 + (0.9)^4 + (0.9)^5 + (0.9)^6\}$$

$$P(2 \leq Y \leq 8) = 0.4695$$

Reasoning and communication

13 $P(\text{even}) = 0.5$

a $P(\text{exactly } 2) = 0.5 \times 0.5 = 0.25$

b $P(\text{at least } 2) = P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$
 $= 1 - 0.5 - 0.25 = 0.25$

c $P(\text{no more than } 2) = P(X = 0) + P(X = 1) + P(X = 2)$
 $= 0.5 + 0.5 \times 0.5 + 0.5 \times 0.5 \times 0.5$
 $= 0.875$

14 $P(\text{sinks putt}) = \frac{3}{4}$

$$P(\text{qqqqp}) = \left(\frac{1}{4}\right)^4 \times \frac{3}{4} = 0.00293$$

15 $P(\text{speed}) = 0.1$

$$P(\text{qqqqqp}) = (0.9)^5 \times 0.1 = 0.059$$

16 $P(\text{fraud}) = 0.25$

a $P(\text{qqqqq}) = 0.75^5 = 0.237$

b $E(X) = \frac{1-p}{p} = \frac{1-0.25}{0.25} = 3$

i.e., you would get away with two and the third one would be expected to be detected.

17 $P(\text{job}) = 0.75$

$$P(\text{qqqp}) = (0.25)^3 0.75 \approx 0.0117$$

Exercise 5.03 The binomial distribution

Concepts and techniques

- 1** **D** ‘the number of 9s in a randomly selected set of 10 digits from a table of random digits’ is binomial as you know how many are 9s and how many are not 9s.
- 2** **B** The binomial distribution with $n = 6$ and $p = 0.5$.
- 3** **E** $4\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^1$ as ${}^4C_3 = 4$ and we want 3 successes $\frac{5}{6}$, and one failure $\frac{1}{6}$.
- 4** **b, c, e** and **g** as there are a fixed number of Bernoulli trials with the same probabilities of success
- 5** **a** $\binom{6}{4}(0.7)^4(0.3)^2 = 0.3241$
- b** $\binom{9}{3}(0.38)^3(0.62)^6 = 0.2618$
- c** $\binom{5}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3 = 0.2048$
- d** $\binom{8}{7}(0.25)^7(0.75)^1 = 0.0004$
- e** $\binom{10}{0}(0.09)^0(0.91)^{10} = 0.3894$
- 6** **a** $\binom{10}{2}(0.5)^2(0.5)^8$ $p = 0.5, q = 0.5, n = 10, x = 2$
- b** $\binom{20}{0}(0.85)^{20}$ $p = 0.15, q = 0.85, n = 20, x = 0$
- c** $\binom{15}{12}\left(\frac{3}{5}\right)^{12}\left(\frac{2}{5}\right)^3$ $p = \frac{3}{5}, q = \frac{2}{5}, n = 15, x = 12$
- d** $\binom{9}{8}(0.11)^8(0.89)$ $p = 0.11, q = 0.89, n = 9, x = 8$
- e** $\binom{7}{4}(0.25)^4(0.75)^3$ $p = 0.25, q = 0.75, n = 7, x = 4$

7 $P(X = x) = \binom{7}{x} (0.8)^x (0.2)^{7-x}$.

- a** The number of trials = 7
b The probability of success in any trial is 0.8
c

x	0	1	2	3	4	5	6	7
P(X = x)	0.0000	0.0004	0.0043	0.0287	0.1147	0.2753	0.3670	0.2097

8 $P(Z = z) = \binom{7}{z} (0.15)^z (0.85)^{7-z}$.

- a** The number of trials is 7.
b The probability of success in any trial is 0.15.
c

z	0	1	2	3	4	5	6	7
P(Z = z)	0.3206	0.3960	0.2097	0.0617	0.0109	0.0012	0.0001	0.0000

9 $n = 7, x = 3, p = \frac{1}{6}, q = \frac{5}{6}$

$$P(X = 3) = {}^7C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^4 = 0.0781$$

Reasoning and communication

- 10** The value of p in question **7** is much higher than the value of p in question **8** and the probabilities of more successes in question **7** are much higher than the corresponding probabilities in question **8**. Similarly, the probabilities of more failures in question **7** are much lower than the corresponding probabilities in question **8**.

Exercise 5.04 Using the binomial distribution

Concepts and techniques

1 **D** $P(X = 5) = {}^6C_5 \left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right)^1 = 0.401\ 877$

2 **B** Binomial probability distribution with $n = 7$ and $p = \frac{2}{5}$.

$$\begin{aligned}P(X \text{ is at least } 6) &= P(X = 6) + P(X = 7) \\ &= 0.0172 + 0.0016 \\ &= 0.0188\end{aligned}$$

3 **D** $p = 0.4, n = 5, P(\text{at most one}) = P(X = 0) + P(X = 1)$

$$\begin{aligned}&= 0.0778 + 0.2592 \\ &= 0.337\end{aligned}$$

4 **D** $n = 8, p = \frac{1}{3}, P(X \geq 1) = 1 - P(X = 0) = 1 - 0.039 = 0.9610$

5 **B** $n = 10, p = 0.75, P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$$\begin{aligned}&= 0.000\ 0010 + 0.000\ 0286 + 0.000\ 3862 \\ &= 0.000\ 4158\end{aligned}$$

6 $n = 7, p = 0.4$

a $P(X = 3) = 0.2903$

b $P(X \text{ is at least } 3) = P(X = 3) + P(X = 4) + \dots + P(X = 7) = 1 - 0.419\ 904 = 0.5801$

c $P(X \text{ is more than } 5) = P(X = 6) + P(X = 7) = 1 - 0.998\ 115\ 84 = 0.018\ 8416$

TI-Nspire CAS

Expression	Result
$\text{binomPdf}(11, 0.82, 7)$	0.086358
$\text{binomCdf}(11, 0.82, 3, 6)$	0.033379
$\text{binomCdf}(11, 0.82, 0, 3)$	0.000108
$1 - \text{binomCdf}(11, 0.82, 0, 7)$	0.880256

ClassPad

Expression	Result
$\text{binomialPDF}(7, 11, 0.82)$	0.08635767214
$\text{binomialCDF}(3, 6, 11, 0.82)$	0.03337855053
$\text{binomialCDF}(0, 3, 11, 0.82)$	$1.07919091 \times 10^{-4}$
$1 - \text{binomialCDF}(0, 7, 11, 0.82)$	0.8802561131

$$n = 11, p = 0.82$$

a $P(X = 7) = 0.0864$

b $P(3 \leq X \leq 6) = P(X \leq 6) - P(X \leq 2) = 0.033386 - 0.000008 = 0.03338$

c $P(X \leq 3) = 0.0001079$

d $P(X > 7) = 1 - P(X \leq 7) = 1 - 0.119744 = 0.8803$

TI-Nspire CAS

The image shows a TI-Nspire CAS calculator screen with the following data:

Expression	Result
$\text{binomPdf}(15, 0.27, 11)$	0.000215
$1 - \text{binomCdf}(15, 0.27, 0, 4)$	0.381003
$1 - \text{binomCdf}(15, 0.27, 0, 8)$	0.007302
$\text{binomCdf}(15, 0.27, 3, 8)$	0.806389

ClassPad

The image shows a ClassPad calculator screen with the following data:

Expression	Result
$\text{binomialPDF}(11, 15, 0.27)$	$2.154891944\text{E-}4$
$1 - \text{binomialCDF}(0, 4, 15, 0.27)$	0.3810027423
$1 - \text{binomialCDF}(0, 8, 15, 0.27)$	$7.30204791\text{E-}3$
$\text{binomialCDF}(3, 8, 15, 0.27)$	0.8063887148

$$n = 15, p = 0.27$$

- a** $P(X = 11) = 0.0002$
- b** $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.618998 = 0.381$
- c** $P(X \geq 9) = 1 - P(X \leq 8) = 0.0073$
- d** $P(2 < X < 9) = P(X \leq 8) - P(X \leq 2) = 0.9927 - 0.1863 = 0.8064$

TI-Nspire CAS

The image shows a TI-Nspire CAS calculator screen with the following data:

Expression	Result
$\text{binomPdf}(25, 0.725, 15)$	0.06498
$\text{binomCdf}(25, 0.725, 5, 15)$	0.1216
$1 - \text{binomCdf}(25, 0.725, 0, 13)$	0.97699
$\text{binomCdf}(25, 0.725, 0, 9)$	0.00015

ClassPad

The image shows a ClassPad calculator screen with the following data:

Expression	Result
$\text{binomialPDF}(15, 25, 0.725)$	0.06498005457
$\text{binomialCDF}(5, 15, 25, 0.725)$	0.1216001154
$1 - \text{binomialCDF}(0, 13, 25, 0.725)$	0.9769903059
$\text{binomialCDF}(0, 9, 25, 0.725)$	$1.499840156 \times 10^{-4}$

$$n = 25, p = 0.725$$

- a $P(X = 15) = 0.0650$
- b $P(5 \leq X \leq 15) = P(X \leq 15) - P(X \leq 4) = 0.1216$
- c $P(X \geq 14) = 1 - P(X \leq 13) = 0.9770$
- d $P(X < 10) = P(X \leq 9) = 0.0001$

10 $n = 8, p = 0.4$

- a $P(2 \text{ successes occur}) = 0.2090$
- b $P(5 \text{ successes occur}) = 0.1239$
- c $P(\text{at least 2 successes occur}) = P(X \geq 2) = 1 - P(X \leq 1) = 0.8936$

TI-Nspire CAS

The image shows a TI-Nspire CAS window with the title '*Unsaved'. The interface displays three rows of calculations:

$\text{binomPdf}(17, 0.65, 5)$	0.002426
$\text{binomCdf}(17, 0.65, 4, 8)$	0.099293
$1 - \text{binomCdf}(17, 0.65, 0, 13)$	0.10279

ClassPad

The image shows a ClassPad interface with a toolbar containing icons for 'Edit', 'Action', and 'Interactive'. Below the toolbar, the following calculations are displayed:

$\text{binomialPDF}(5, 17, 0.65)$	$2.426239229\text{E-}3$
$\text{binomialCDF}(4, 8, 17, 0.65)$	0.09929265161
$1 - \text{binomialCDF}(0, 13, 17, 0.65)$	0.1027901803

$$n = 17, p = 0.65$$

- a** $P(X = 5) = 0.0024$
- b** $P(4 \leq X \leq 8) = P(X \leq 8) - P(X \leq 3) = 0.0993$
- c** $P(X \geq 14) = 1 - P(X \leq 13) = 0.1028$

Reasoning and communication

12 $n = 6, p = 0.5$

a $P(X = x) = 0.3125$
 $= {}^6C_x (0.5)^x (0.5)^{6-x}$
 $x = 3$

b $P(X = x) = 0.09375$
 $x = 1, 5$

c $P(X = x) = 0.234375$
 $x = 2, 4$

13 $n = 5, q = 0.5$

a $P(X < x) = 0.1875$
 $x = 2$ as $P(X \leq 1) = 0.1875$

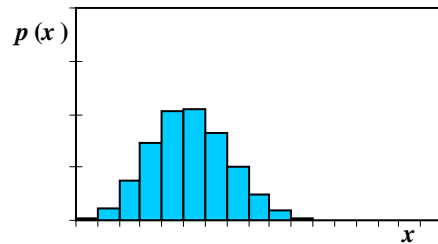
b $P(X \geq x) = 0.1875$
 $P(X \leq x - 1) = 0.8125$
 $x - 1 = 3$, so $x = 4$

c $P(X > x) = 0.5$
 $P(X \leq x) = 0.5$
 $x = 2$

Exercise 5.05 Properties of the binomial distribution

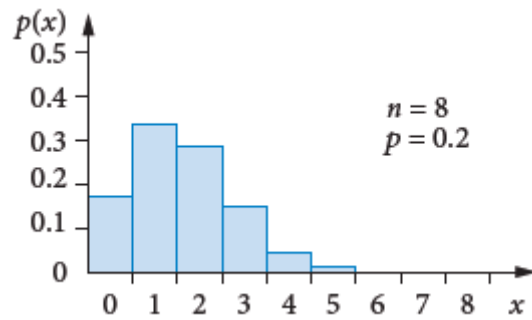
Concepts and techniques

1 C as clustered around 0.3

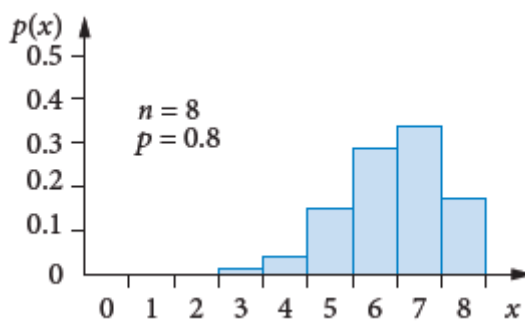


2 D $n = 32$ and $p = \frac{1}{4}$
 $\mu = np = 8, \sigma^2 = npq = 6$

3 a $n = 8$ and $p = 0.2$

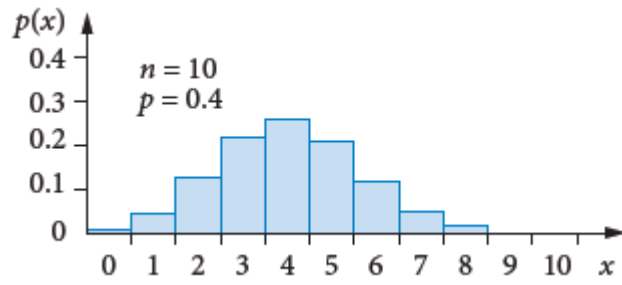


$n = 8$ and $p = 0.8$

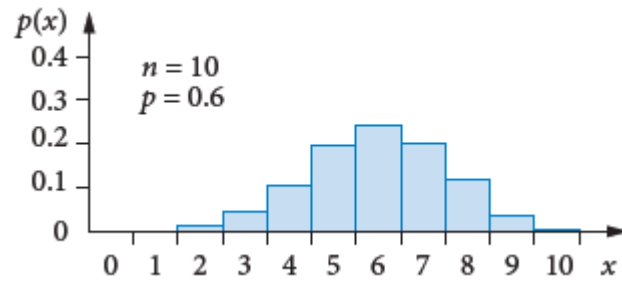


b The graphs are mirror images of each other, skewed positively and negatively respectively.

4 a $n = 10$ and $p = 0.4$

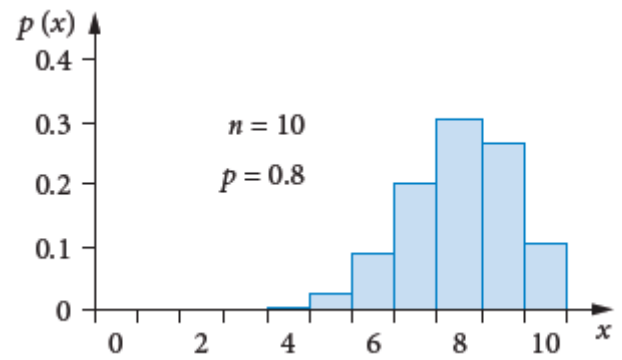


$n = 10$ and $p = 0.6$



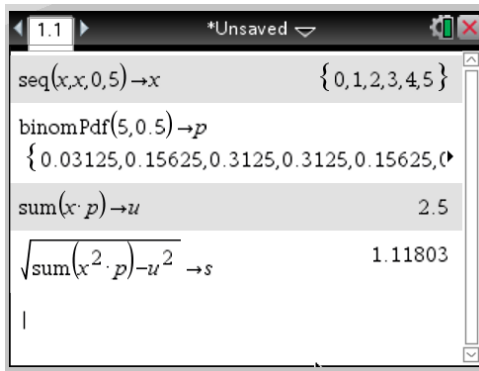
b The graphs are mirror images of each other, skewed positively and negatively respectively.

5 a

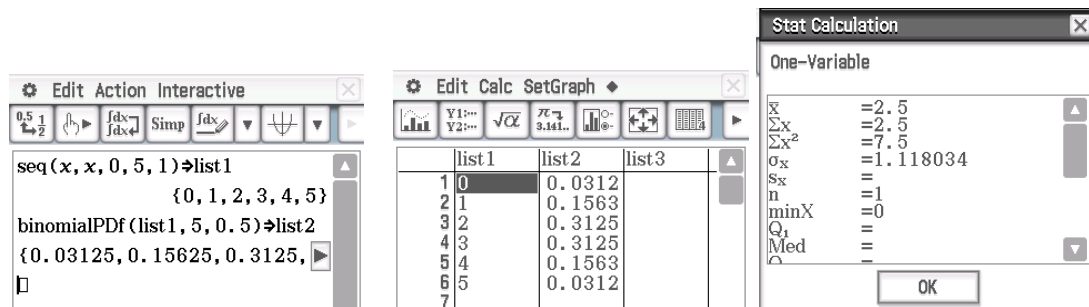


b 8

TI-Nspire CAS



ClassPad



a $n = 5$ and $p = 0.5$

x	0	1	2	3	4	5	Sums
$P(X = x)$	0.0313	0.1563	0.3125	0.3125	0.1563	0.0313	1.0002
$x \times P(X = x)$	0	0.1563	0.625	0.9375	0.6252	0.1565	2.5005
$x^2 \times P(X = x)$	0	0.1563	1.25	2.8125	2.5008	0.7825	7.5021

$$\begin{aligned} \mu = E(X) &= 0 \times 0.0313 + 1 \times 0.1563 + 2 \times 0.3125 + 3 \times 0.3125 \\ &\quad + 4 \times 0.1563 + 5 \times 0.0313 = 2.5005 \end{aligned}$$

$$\begin{aligned} E(X^2) &= 0^2 \times 0.0313 + 1^2 \times 0.1563 + 2^2 \times 0.3125 + 3^2 \times 0.3125 \\ &\quad + 4^2 \times 0.1563 + 5^2 \times 0.0313 = 7.5021 \end{aligned}$$

$$\sigma_x^2 = E(X^2) - \mu^2$$

$$\sigma_x^2 = 7.5021 - 2.5005^2$$

$$= 1.2496$$

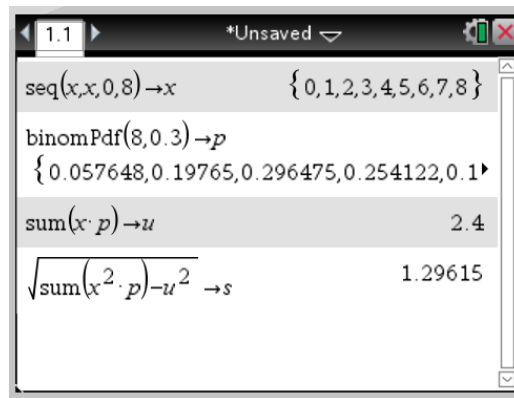
$$\sigma_x \approx 1.118$$

$$\mu = np = 5 \times 0.5 = 2.5$$

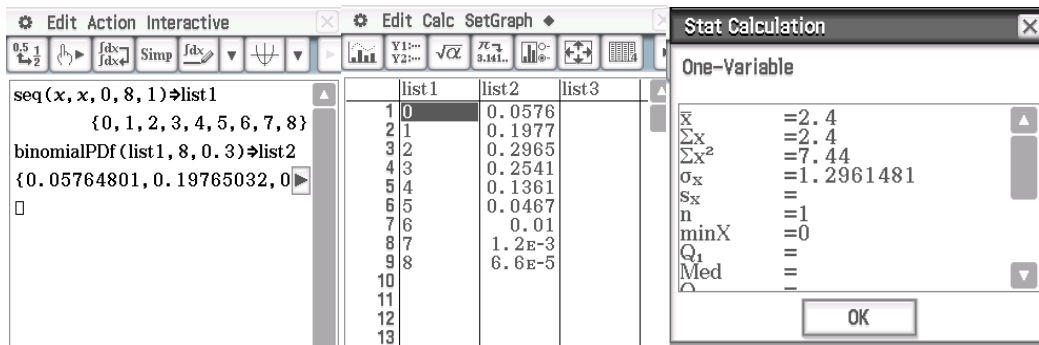
$$\sigma = \sqrt{npq} = \sqrt{5 \times 0.5 \times 0.5} = 1.118$$

b

TI-Nspire CAS



ClassPad



$n = 8$ and $p = 0.3$

x	0	1	2	3	4	5	6	7	8	Sums
$P(X = x)$	0.0576	0.1977	0.2965	0.2541	0.1361	0.0467	0.01	0.0012	0.0001	1
$x \times P(X = x)$	0	0.1977	0.593	0.7623	0.5444	0.2335	0.06	0.0084	0.0005	2.3998
$x^2 \times P(X = x)$	0	0.1977	1.186	2.2869	2.1776	1.1675	0.36	0.0588	0.0025	7.437

$$\mu = E(X) = 2.3998$$

$$E(X^2) = 7.437$$

$$\sigma_x^2 = E(X^2) - \mu^2$$

$$\sigma_x^2 = 7.437 - 2.3998^2$$

$$= 1.68$$

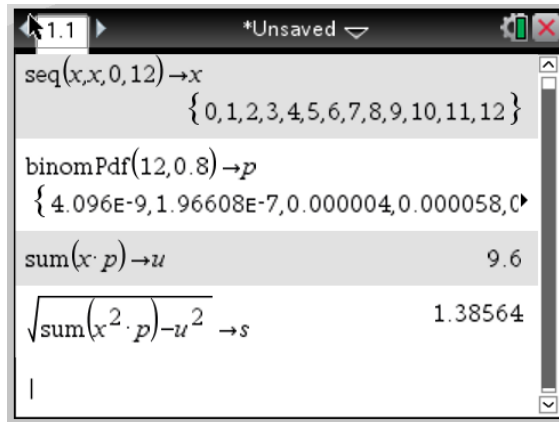
$$\sigma_x \approx 1.296$$

$$\mu = np = 8 \times 0.3 = 2.4$$

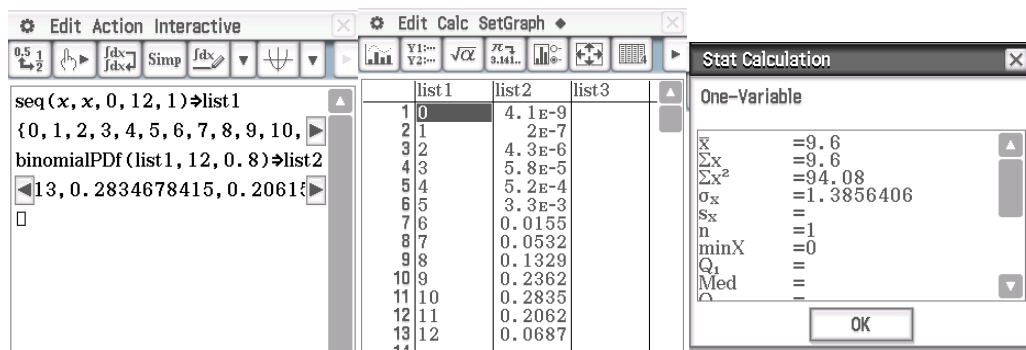
$$\sigma = \sqrt{npq} = \sqrt{8 \times 0.3 \times 0.7} = 1.296$$

c

TI-Nspire CAS



ClassPad



$n = 12$ and $p = 0.8$

x	0	1	2	3	4	5	6
$P(X = x)$	0	0	0	0	0	0.0033	0.0155
$x \times P(X = x)$	0	0	0	0	0	0.0165	0.093
$x^2 \times P(X = x)$	0	0	0	0	0	0.0825	0.558

	7	8	9	10	11	12	Sums
	0.0532	0.1329	0.2362	0.2835	0.2062	0.0687	0.9995
	0.3724	1.0632	2.1258	2.835	2.2682	0.8244	9.5985
	2.6068	8.5056	19.1322	28.35	24.9502	9.8928	94.0781

$$\mu = E(X) = 9.5985$$

$$E(X^2) = 94.0781$$

$$\sigma_x^2 = E(X^2) - \mu^2$$

$$\begin{aligned} \sigma_x^2 &= 94.0781 - 9.5985^2 \\ &= 1.9469 \end{aligned}$$

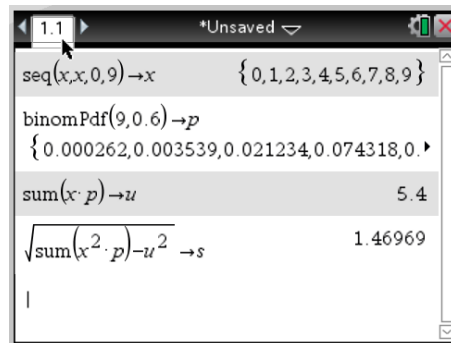
$$\sigma_x \approx 1.39$$

$$\mu = np = 12 \times 0.8 = 9.6$$

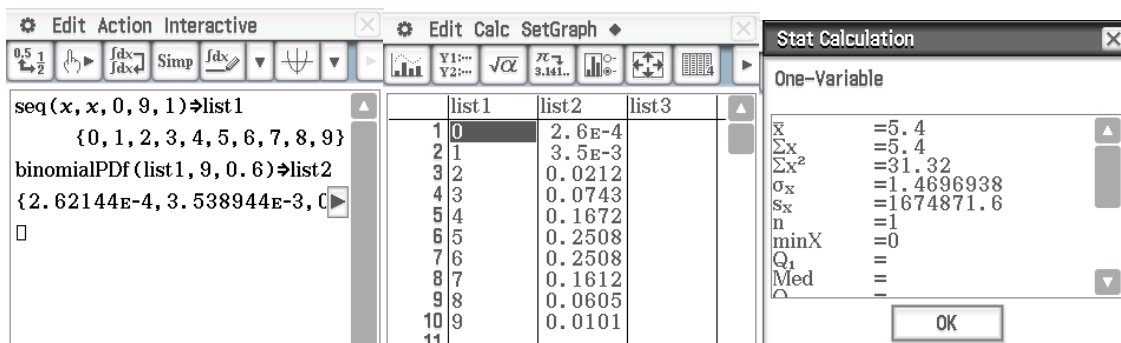
$$\sigma = \sqrt{npq} = \sqrt{12 \times 0.8 \times 0.2} = 1.39$$

d

TI-Nspire CAS



ClassPad



$n = 9$ and $p = 0.6$

x	0	1	2	3	4	5	6	7	8	9	Sums
$P(X = x)$	0.0003	0.0035	0.0212	0.0743	0.1672	0.2508	0.2508	0.1612	0.0605	0.0101	0.9999
$x \times P(X = x)$	0	0.0035	0.0424	0.2229	0.6688	1.254	1.5048	1.1284	0.484	0.0909	5.3997
$x^2 \times P(X = x)$	0	0.0035	0.0848	0.6687	2.6752	6.27	9.0288	7.8988	3.872	0.8181	31.3199

$$\mu = E(X) = 5.4$$

$$E(X^2) = 31.32$$

$$\sigma_x^2 = E(X^2) - \mu^2$$

$$\sigma_x^2 = 31.32 - 5.4^2$$

$$= 2.16$$

$$\sigma_x \approx 1.47$$

$$\mu = np = 9 \times 0.6 = 5.4$$

$$\sigma = \sqrt{npq} = \sqrt{9 \times 0.6 \times 0.4} = 1.47$$

7 a $n = 7$ and $p = 0.1$

$$\mu = np = 7 \times 0.1 = 0.7$$

$$\sigma = \sqrt{npq} = \sqrt{7 \times 0.1 \times 0.9} = 0.794$$

b $n = 7$ and $p = 0.9$

$$\mu = np = 7 \times 0.9 = 6.3$$

$$\sigma = \sqrt{npq} = \sqrt{7 \times 0.9 \times 0.1} = 0.794$$

c $n = 20$ and $p = 0.65$

$$\mu = np = 20 \times 0.65 = 13$$

$$\sigma = \sqrt{npq} = \sqrt{20 \times 0.65 \times 0.35} = 2.133$$

d $n = 30$ and $p = 0.34$

$$\mu = np = 30 \times 0.34 = 10.2$$

$$\sigma = \sqrt{npq} = \sqrt{30 \times 0.34 \times 0.66} = 2.59$$

Reasoning and communication

8 $n = 10, \text{Var}(X) = 0.9 = npq$

$$0.9 = 10p(1 - p)$$

$$p = 0.1 \text{ or } p = 0.9$$

9 A $\mu = np = 24, SD(X) = 3 = \sqrt{npq}$

$$3 = \sqrt{24(1 - p)}$$

$$9 = 24(1 - p)$$

$$p = 0.625 = \frac{5}{8}$$

10 a Binomial as you either have an ace or NOT an ace.

b $\mu = np = 60 \times \frac{4}{52} = \frac{60}{13} = 4.62$

Expect about 5.

c About 95%.

11 $\mu = 5, \sigma = 2$

$$\mu = np = 5$$

$$2 = \sqrt{npq}$$

$$4 = 5q$$

$$q = 0.8, p = 0.2, n = \frac{5}{0.2} = 25$$

$$P(X = 5) = 0.196$$

12 $\mu = np = 200 \times 0.5 = 100$

13 $p = 0.8, \mu = 500, n = ?$

$$\mu = np$$

$$500 = n \times 0.8$$

$$n = 625$$

14 a $n = 420$ batteries, $\mu = np = 420 \times 0.4 = 168$

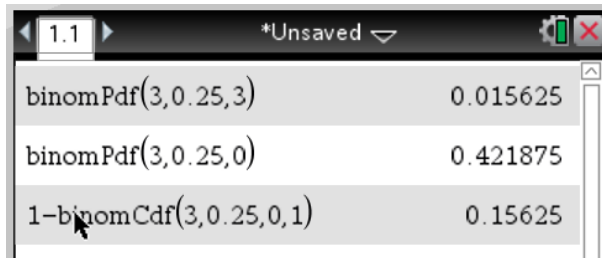
b $\sigma = \sqrt{npq} = \sqrt{420 \times 0.4 \times 0.6} = 10.04$

Exercise 5.06 Applications of the binomial distribution

Concepts and techniques

1

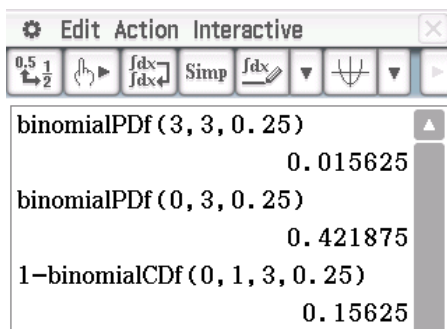
TI-Nspire CAS



The image shows a TI-Nspire CAS calculator screen with a table of results. The table has two columns: the first column contains binomial distribution functions and the second column contains their numerical values. The functions are: binomPdf(3, 0.25, 3), binomPdf(3, 0.25, 0), and 1-binomCdf(3, 0.25, 0, 1). The values are 0.015625, 0.421875, and 0.15625 respectively.

$\text{binomPdf}(3, 0.25, 3)$	0.015625
$\text{binomPdf}(3, 0.25, 0)$	0.421875
$1 - \text{binomCdf}(3, 0.25, 0, 1)$	0.15625

ClassPad



The image shows a ClassPad calculator screen with a table of results. The table has two columns: the first column contains binomial distribution functions and the second column contains their numerical values. The functions are: binomialPDF(3, 3, 0.25), binomialPDF(0, 3, 0.25), and 1-binomialCDF(0, 1, 3, 0.25). The values are 0.015625, 0.421875, and 0.15625 respectively.

$\text{binomialPDF}(3, 3, 0.25)$	0.015625
$\text{binomialPDF}(0, 3, 0.25)$	0.421875
$1 - \text{binomialCDF}(0, 1, 3, 0.25)$	0.15625

- a** $n = 3, p = 0.25$ and $q = 0.75$
- b** $P(x = 3) = 0.015625$
- c** $P(x = 0) = 0.421875$
- d** $P(\text{at least 2 flowers are red}) = 1 - P(x \leq 1) = 0.15625$

TI-Nspire CAS

The image shows a TI-Nspire CAS calculator window titled "1.1" and "*Unsaved". It displays three rows of calculations:

$\text{binomPdf}\left(4, \frac{2}{3}, 2\right)$	0.296296
$1 - \text{binomCdf}\left(4, \frac{2}{3}, 0, 0\right)$	0.987654
$1 - \text{binomCdf}\left(4, \frac{2}{3}, 0, 2\right)$	0.592593

ClassPad

The image shows a ClassPad calculator interface with a toolbar containing icons for "Edit", "Action", and "Interactive". The main display area shows three rows of calculations:

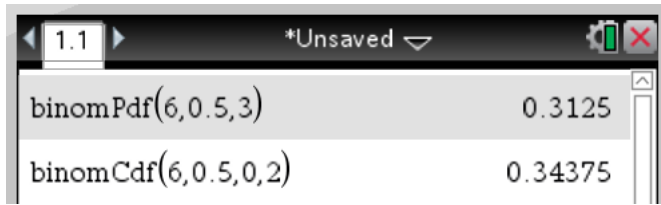
$\text{binomialPDF}\left(2, 4, \frac{2}{3}\right)$	0.2962962963
$1 - \text{binomialPDF}\left(0, 4, \frac{2}{3}\right)$	0.987654321
$\text{binomialCDF}\left(3, 4, 4, \frac{2}{3}\right)$	0.5925925926

$$n = 4, p = \frac{2}{3}$$

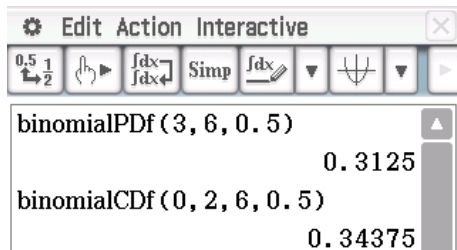
- a** $P(x = 2) = 0.296$
- b** $P(x \geq 1) = 1 - P(x = 0) = 0.988$
- c** $P(x > 2) = 1 - P(x \leq 2) = 0.593$

3

TI-Nspire CAS



ClassPad



$n = 6$ children, $p = 0.5$

a $P(3 \text{ boys and } 3 \text{ girls}) = 0.3125$

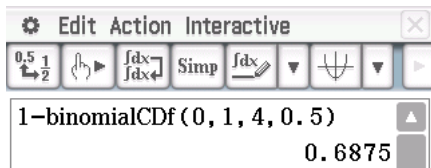
b $P(x = 0 \text{ or } 1 \text{ or } 2) = P(x \leq 2) = 0.34375$

4

TI-Nspire CAS



ClassPad



$n = 4$ children, $p = 0.5$

$P(x \geq 2) = 1 - P(x \leq 1) = 0.6875$

5 TI-Nspire CAS

binomPdf(7,0.15,7)	0.000002
1-binomCdf(7,0.15,0,0)	0.679423

ClassPad

binomialPDF(7, 7, 0.15)	1.70859375E-6
1-binomialPDF(0, 7, 0.15)	0.6794229117

$n = 7, p = 0.15$

a $P(x = 7) = 0.000\ 001\ 7$

b $P(x \geq 1) = 1 - P(x = 0) = 0.679$

6 TI-Nspire CAS

binomPdf(8,0.05,0)	0.66342
binomPdf(8,0.05,2)	0.051456
1-0.66342	0.33658

ClassPad

binomialPDF(0, 8, 0.05)	0.6634204313
binomialPDF(2, 8, 0.05)	0.05145643234
1-0.6634204313	0.3365795687

$n = 8, p = 0.05$

a $P(x = 0) = 0.663$

b $P(x = 2) = 0.051$

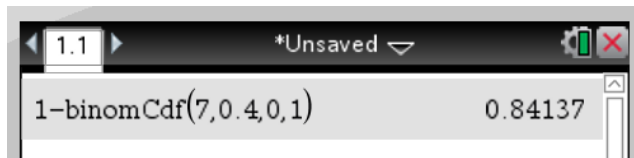
c $P(x \geq 1) = 1 - P(x = 0) = 0.337$

7 a $(0.4)^2(0.6)^5 = 0.012$

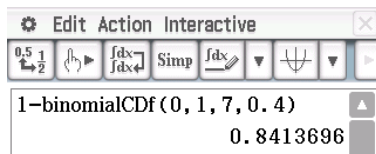
b $(0.4)^2 = 0.16$

c

TI-Nspire CAS



ClassPad



$$n = 7, p = 0.4, P(x \geq 2) = 1 - P(x \leq 1) = 0.84$$

TI-Nspire CAS

The image shows a TI-Nspire CAS window titled '*Unsaved'. It displays a list of four binomial distribution calculations and their results:

$\text{binomPdf}(7, 0.25, 7)$	0.000061
$\text{binomPdf}(7, 0.25, 0)$	0.133484
$\text{binomPdf}(7, 0.25, 3)$	0.173035
$1 - \text{binomCdf}(7, 0.25, 0, 2)$	0.243591

ClassPad

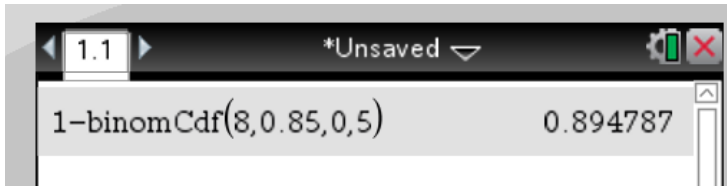
The image shows a ClassPad interface window titled 'Edit Action Interactive'. It displays a list of four binomial distribution calculations and their results:

$\text{binomialPDF}(7, 7, 0.25)$	$6.103515625\text{E-}5$
$\text{binomialPDF}(0, 7, 0.25)$	0.1334838867
$\text{binomialPDF}(3, 7, 0.25)$	0.173034668
$1 - \text{binomialCDF}(0, 2, 7, 0.25)$	0.2435913086

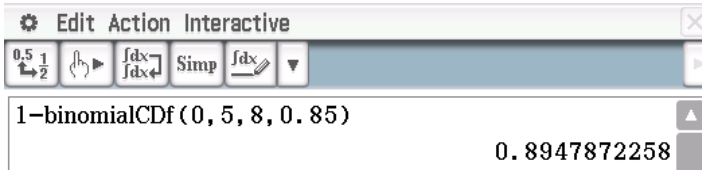
$$n = 7, p = 0.25$$

- a** $P(x = 7) = 0.000\ 062$
- b** $P(x = 0) = 0.133$
- c** $P(x = 3) = 0.173$
- d** $P(x \geq 3) = 1 - P(x \leq 2) = 0.243$

9 TI-Nspire CAS



ClassPad



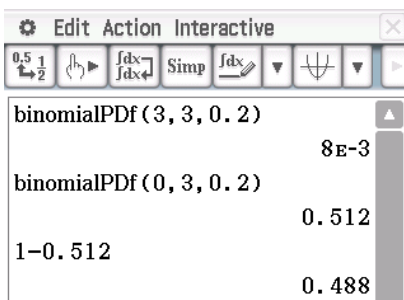
$n = 8, p = 0.85$

$P(x \geq 6) = 1 - P(x \leq 5) = 0.895$

10 TI-Nspire CAS



ClassPad



a $n = 3, p = 0.2, q = 0.8$

b $P(x = 3) = 0.008$

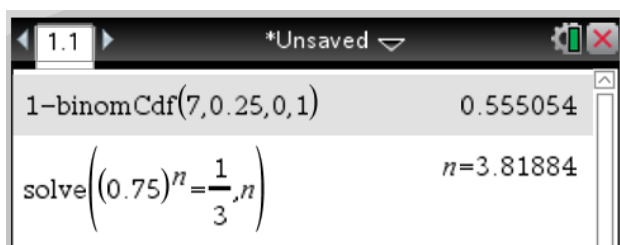
c $P(x = 0) = 0.512$

d $P(x \geq 1) = 1 - P(x = 0) = 0.488$

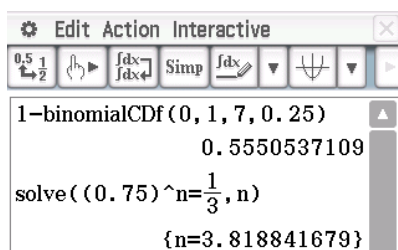
Reasoning and communication

11

TI-Nspire CAS



ClassPad



$$p = \frac{1}{4}$$

a $n = 7, P(X \geq 2) = 1 - P(X \leq 1) = 0.555$

b $n = ?, P(X \geq 1) > \frac{2}{3}$

$$P(X \geq 1) = 1 - P(x = 0) > \frac{2}{3}$$

$$\text{i.e. } P(x = 0) < \frac{1}{3}$$

$$\Rightarrow \binom{n}{0} \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^n < \frac{1}{3}$$

$$\Rightarrow \left(\frac{3}{4}\right)^n < \frac{1}{3}$$

$$n = 4$$

TI-Nspire CAS

The image shows a TI-Nspire CAS calculator window with the title '*Unsaved'. The window contains two rows of calculations:

$\text{binomPdf}(6, 0.63, 3)$	0.253313
$1 - \text{binomCdf}(6, 0.63, 0, 2)$	0.859644

ClassPad

The image shows a ClassPad calculator window with the title 'Edit Action Interactive'. The window contains two rows of calculations:

$\text{binomialPDF}(3, 6, 0.63)$	0.2533126138
$1 - \text{binomialCDF}(0, 2, 6, 0.63)$	0.8596440827

$$n = 6, p = 0.63$$

a $P(x = 3) = 0.253$

b $P(x \geq 3) = 1 - P(x \leq 2) = 0.860$

TI-Nspire CAS

Expression	Result
$1500 \cdot 0.68 \rightarrow \mu$	1020.
$\sqrt{1500 \cdot 0.68 \cdot 0.32} \rightarrow \sigma$	18.0665
$\mu - 2 \cdot \sigma$	983.867
$\mu + 2 \cdot \sigma$	1056.13
$\text{binomCdf}(1500, 0.68, 984, 1056)$	0.956695

ClassPad

Expression	Result
$1500 \times 0.68 \rightarrow \mu$	1020
$\sqrt{1500 \times 0.68 \times 0.32} \rightarrow \sigma$	18.06654367
$\mu - 2\sigma$	983.8669127
$\mu + 2\sigma$	1056.133087
$\text{binomialCDF}(984, 1056, 1500, \rightarrow)$	0.9566948303

$$n = 1500, p = 0.68$$

$$\mu = np = 1500 \times 0.68 = 1020$$

$$\sigma = \sqrt{npq} = \sqrt{1500 \times 0.68 \times 0.32} = 18.066\dots$$

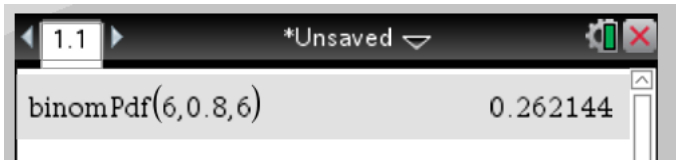
$$2\sigma \approx 36.133\dots$$

$$\mu + 2\sigma = 1056, \mu - 2\sigma = 984$$

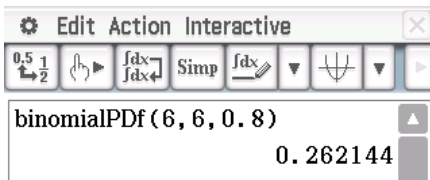
There is a 95% chance that between 984 and 1056 students will attend a government school.

14

TI-Nspire CAS



ClassPad



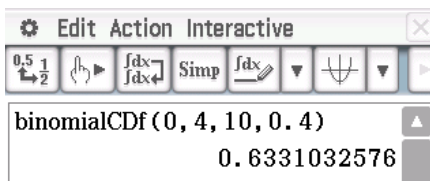
$$n = 6, p = 0.8, P(x = 6) = 0.262$$

15

TI-Nspire CAS



ClassPad

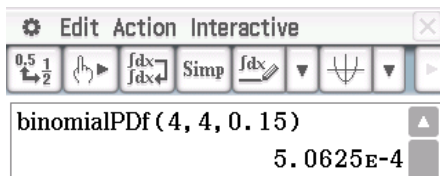


$$n = 10, p = 0.4, P(x < 5) = P(x \leq 4) = 0.633$$

TI-Nspire CAS

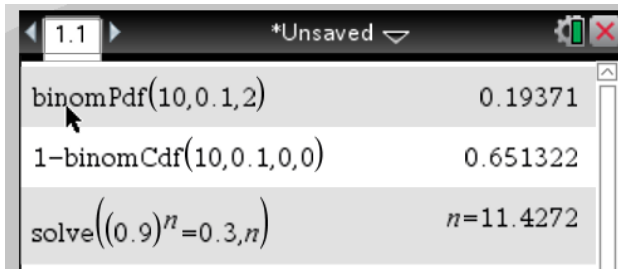


ClassPad

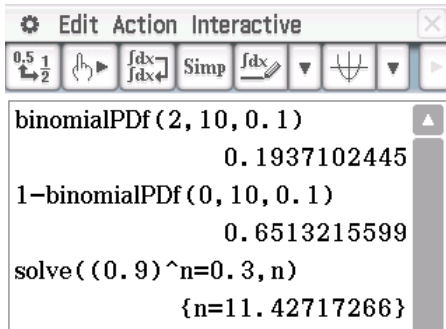


$$n = 4, p = 0.15, P(x = 4) = 0.0005$$

17 TI-Nspire CAS



ClassPad



$p = 0.1$

a $n = 10$

i $P(x = 2) = 0.194$

ii $P(x \geq 1) = 1 - P(x = 0) = 0.651$

b $n = ?$

$P(x \geq 1) > 0.7$

$1 - P(x = 0) > 0.7$

$P(x = 0) < 0.3$

${}^n C_0 (0.1)^0 (0.9)^n < 0.3$

i.e. $(0.9)^n < 0.3$

$n \log_e (0.9) < \log_e (0.3)$

[Note: $\log_e (0.9) < 0$]

$$n > \frac{\log_e (0.3)}{\log_e (0.9)} = 11.42$$

$n = 12$

TI-Nspire CAS

The image shows a TI-Nspire CAS window with a title bar containing "1.1" and "*Unsaved". The main area displays two rows of calculations:

$\text{binomPdf}(8, 0.2, 3)$	0.146801
$1 - \text{binomCdf}(8, 0.2, 0, 0)$	0.832228

ClassPad

The image shows a ClassPad interface with a title bar "Edit Action Interactive". Below the title bar is a toolbar with various mathematical symbols and functions. The main area displays two rows of calculations:

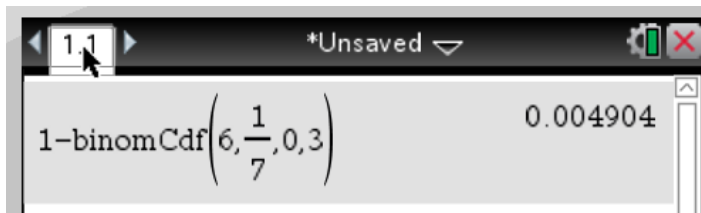
$\text{binomialPDF}(3, 8, 0.2)$	0.14680064
$1 - \text{binomialPDF}(0, 8, 0.2)$	0.83222784

$$p = 0.2, n = 8$$

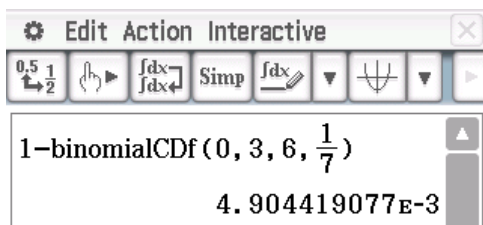
a $P(x = 3) = 0.147$

b $P(x \geq 1) = 1 - P(x = 0) = 0.832$

TI-Nspire CAS



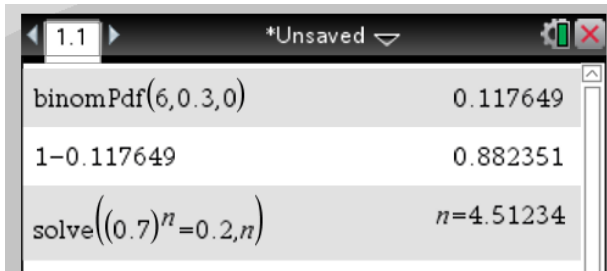
ClassPad



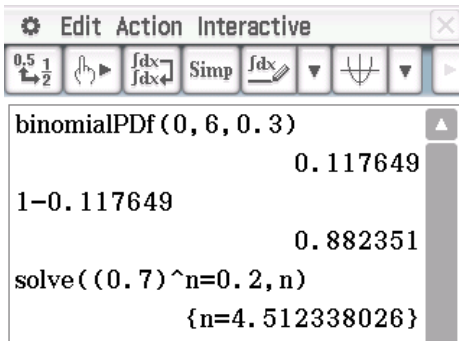
$$p = \frac{1}{7}, n = 6$$

$$P(x \geq 4) = 1 - P(x \leq 3) = \frac{577}{117\,649}$$

20 TI-Nspire CAS



ClassPad



$p = 0.3$

- a** $n = 6$
 - i** $P(x = 0) = 0.1176$
 - ii** $P(x \geq 1) = 1 - P(x = 0) = 0.882$

- b** $n = ?$
 - $P(x \geq 1) > 0.8$
 - $1 - P(x = 0) > 0.8$
 - $P(x = 0) < 0.2$
 - ${}^n C_0 (0.7)^n (0.3)^0 < 0.2$
 - i.e. $(0.7)^n < 0.2$
 - $n \log_e (0.7) < \log_e (0.2)$
 - [Note: $\log_e (0.7) < 0$]
 - $n > \frac{\log_e (0.2)}{\log_e (0.7)} = 4.51$
 - $n = 5$

TI-Nspire CAS

Expression	Result
$\text{binomPdf}(10, 0.15, 2)$	0.275897
$(0.15)^3$	0.003375
$1 - \text{binomCdf}(10, 0.15, 0, 2)$	0.179804

ClassPad

Expression	Result
$\text{binomialPDF}(2, 10, 0.15)$	0.2758966566
$1 - \text{binomialCDF}(0, 2, 10, 0.15)$	0.1798035196

$$n = 10, p = 0.15$$

a $P(x = 2) = 0.276$

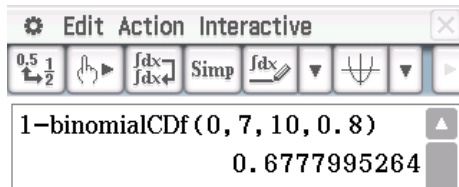
b This is not binomial, $P(\text{first 3 defective}) = (0.15)^3 = 0.003375$

c $P(x \geq 3) = 1 - P(x \leq 2) = 0.18$

TI-Nspire CAS



ClassPad



$$n = 10, p = 0.8$$

$$P(x \geq 8) = 1 - P(x \leq 7) = 0.678$$

Chapter 5 Review

Multiple choice

- 1 B As there is either a six or not a six. Two possible outcomes.
- 2 C As $\sigma = \sqrt{pq} = \sqrt{0.2 \times 0.8} = \sqrt{0.16} = 0.4$.
- 3 D 'the number of cards drawn from a well shuffled deck of playing cards before an ace occurs.' as the sequence is q, q, q, \dots, p
- 4 E $P(X = 3) = (0.6)^3(0.4) = 0.0864$
- 5 A 'the number of hearts that occur when 7 cards are drawn with replacement from a well-shuffled deck of 52 cards' as number of hearts are successes and not hearts are failures.
- 6 E The binomial distribution with $n = 12$ and $p = 0.25$.
- 7 B $n = 7, p = \frac{5}{12}, P(x \leq 1) = 0.1379$
- 8 D $n = 20, p = 0.9, P(x \geq 18) = 1 - P(x \leq 17) = 0.677$
- 9 D $\mu = np = 20 \times 0.7 = 14$

Short answer

- 10 a No as there is more than one suit. Binomial needs success and failure – two options.
- b No, haven't got two alternatives only.
- c Yes, as orange or not orange only.
- d No, there are up to 15 possible tails.
- e Yes, as a king or not a king.
- f No, as there are up to 3 blue marbles.
- g Yes, as either left-handed or not.
- 11 $p = \frac{8}{45}$
- 12 $p = \frac{1}{6}, \sigma^2 = npq = 1 \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$

13 $p = \frac{1}{3}$

$$P(x = 5) = \left(\frac{2}{3}\right)^4 \times \frac{1}{3} = \frac{16}{243}$$

14 $p = 0.3$

a $P(X = 3) = (0.7)^3(0.3) = 0.1029$

b
$$\begin{aligned} P(X \leq 5) &= (0.3) + (0.7)^1(0.3) + (0.7)^2(0.3) + (0.7)^3(0.3) + (0.7)^4(0.3) + (0.7)^5(0.3) \\ &= 0.3 + 0.3 \times 0.7(1 + 0.7 + 0.7^2 + \dots + 0.7^4) \\ &= 0.8824 \end{aligned}$$

c Assuming she keeps going round and round until she gets a win,

$$E(X) = \frac{1-p}{p} = \frac{1-0.3}{0.3} = 2.333\dots$$

15 $p = 0.68$

a $P(Y = 3) = (0.32)^3(0.68) = 0.0223$

b
$$\begin{aligned} P(2 \leq Y \leq 9) &= (0.32)^2(0.68) + (0.32)^3(0.68) + (0.32)^4(0.68) + (0.32)^5(0.68) \\ &\quad + (0.32)^6(0.68) + (0.32)^7(0.68) + (0.32)^8(0.68) + (0.32)^9(0.68) \\ &= (0.32)^2(0.68)[1 + (0.32) + (0.32)^2 + (0.32)^3 + (0.32)^4 + (0.32)^5 + (0.32)^6 + (0.32)^7] \\ &= 0.1024 \end{aligned}$$

16 **a** Yes, there are two alternatives, prime or not prime.

b No, the outcomes vary.

c Yes, binomial. The card is either a spade or it is not.

d No, not binomial. The number of tosses required can vary.

e Yes, binomial. The die shows either an even number or not an even number.

f Without replacement means the probability of obtaining green changes, so not binomial.

g Recording heads may vary. Not binomial.

17 a $\binom{5}{2}(0.4)^2(0.6)^3 = 0.3456$

b $\binom{10}{7}(0.43)^7(0.57)^3 = 0.604$

c $\binom{8}{2}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^6 = 0.00171$

18 a $n = 8$

b $p = 0.45$

c

x	0	1	2	3	4	5	6	7	8
$P(X = x)$	0.0084	0.0548	0.1569	0.2568	0.2627	0.1719	0.0703	0.0164	0.0017

19 $n = 8, p = 0.35$

a $P(X = 7) = 0.0033$

b $P(x \geq 2) = 1 - P(x \leq 1) = 0.831$

c $P(X > 4) = 1 - P(x \leq 4) = 0.106$

20 $n = 5, p = \frac{12}{52}$

$P(X = 2) = 0.242$

21 $n = 20, p = 0.68$

a $P(X = 12) = 0.1354$

b $P(X \geq 7) = 1 - P(x \leq 6) = 0.9994$

c $P(X \leq 9) = 0.0279$

22 $n = 9, p = 0.6$

a $P(x = 3) = 0.0743$

b $P(x = 6) = 0.2508$

c $P(X \geq 2) = 1 - P(x \leq 1) = 0.9962$

23 $n = 3, p = 0.7$

$P(X \geq 2) = 1 - P(x \leq 1) = 0.784$

Expression	Result
$\text{binomPdf}(15, 0.3, 5)$	0.20613
$\text{binomPdf}(19, 0.45, 7)$	0.144267
$\text{binomCdf}(18, 0.75, 0, 5)$	0.000034
$1 - \text{binomCdf}(12, 0.6, 0, 3)$	0.984733
$\text{binomCdf}(20, 0.3, 3, 9)$	0.916555

ClassPad

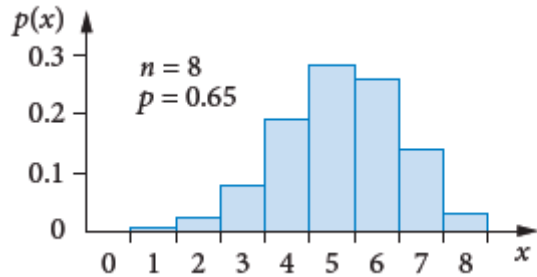
Expression	Result
$\text{binomialPDF}(5, 15, 0.3)$	0.206130381
$\text{binomialPDF}(7, 19, 0.45)$	0.1442669967
$\text{binomialCDF}(0, 5, 18, 0.75)$	$3.42457206 \times 10^{-5}$
$1 - \text{binomialCDF}(0, 3, 12, 0.6)$	0.9847327334
$\text{binomialCDF}(3, 9, 20, 0.3)$	0.9165549704

- a** $n = 15$ and $p = 0.3$
 $P(x = 5) = 0.2061$
- b** $n = 19$ and $p = 0.45$
 $P(x = 7) = 0.1442$
- c** $n = 18$ and $p = 0.75$
 $P(x < 6) = P(x \leq 5) = 0.000\ 034\dots$
- d** $n = 12$ and $p = 0.6$
 $P(x \geq 4) = 1 - P(x \leq 3) = 0.9847$
- e** $n = 20$ and $p = 0.3$
 $P(3 \leq x \leq 9) = P(x \leq 9) - P(x \leq 2) = 0.9166$

25 $n = 12$ and $p = 0.5$

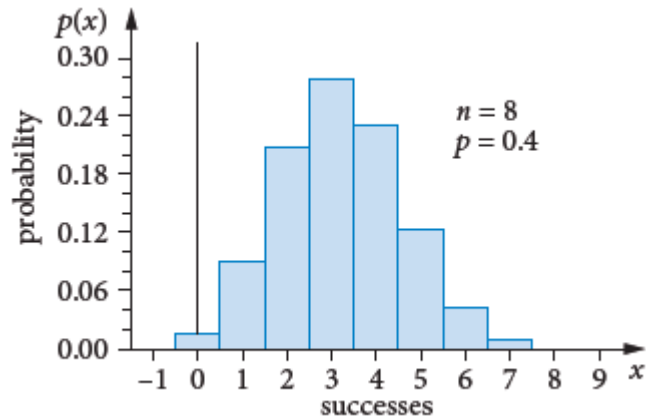
$$P(3 < x < 7) = P(x \leq 6) - P(x \leq 3) = 0.5398$$

26 $n = 8$ and $p = 0.65$



Slightly skewed to the right. i.e., slightly negatively skewed.

27 a $n = 8$ and $p = 0.4$



b The most likely number of successes is 3.

- 28** **a** $n = 8$ and $p = 0.2$
 $\mu = np = 8 \times 0.2 = 1.6$
 $\sigma = \sqrt{npq} = \sqrt{8 \times 0.2 \times 0.8} = \sqrt{1.28} = 1.131$
- b** $n = 10$ and $p = 0.7$
 $\mu = np = 10 \times 0.7 = 7$
 $\sigma = \sqrt{npq} = \sqrt{10 \times 0.7 \times 0.3} = \sqrt{2.1} = 1.45$
- c** $n = 15$ and $p = 0.55$
 $\mu = np = 15 \times 0.55 = 8.25$
 $\sigma = \sqrt{npq} = \sqrt{15 \times 0.55 \times 0.45} = \sqrt{3.7125} = 1.93$
- 29** $n = 20$ and $p = 0.15$
- a** $P(x = 7) = 0.016$
- b** $P(X \geq 7) = 1 - P(x \leq 6) = 0.0219$
- c** $P(X \geq 5) = 1 - P(x \leq 4) = 0.1702$
- 30** $n = 12$ and $p = 0.28$
- a** $(0.28)^2 \times (0.72)^{10} = 0.002\ 935$
- b** $P(X = 2) = 0.1937$
- c** $P(X \geq 2) = 1 - P(x \leq 1) = 0.8900$

Application

- 31** $n = 10$ and $p = 0.8$
- a** $P(X = 10) = 0.1074$
- b** $P(X = 9) = 0.2684$
- c** $P(X \geq 7) = 1 - P(x \leq 6) = 0.8791$
- 32** $n = 15$ and $p = 0.85$
- a** $P(X = 15) = (0.85)^{15} = 0.087\ 354$
- b** Use $q = 0.15$ as success
 $P(\text{at least 2 have no immunity to the disease}) = P(X \geq 2) = 1 - P(x \leq 1) = 0.6814$
- c** Use $q = 0.15$ as success
 $P(X < 4) = P(X \leq 3) = 0.8227$

- 33** $n = 15$ and $p = 0.05$ (defective)
- a** $P(\text{all are operative}) = P(X = 0) = 0.4633$
- b** $P(X = 2) = 0.1348$
- c** $P(X \geq 2) = 1 - P(X \leq 1) = 0.171$
- d** $P(X \leq 2) = 0.9638$
- 34** $n = 18$ and $p = 0.3$
 $P(X \leq 4) = 0.3327$
- 35** $n = 25$ and $p = 0.18$
- a** $\mu = np = 25 \times 0.18 = 4.5$
 4 or 5 are expected to be iron deficient.
- b** $\sigma = \sqrt{npq} = \sqrt{25 \times 0.18 \times 0.82} = 1.92094$
 $\mu + 2\sigma = 4.5 + 2(1.92) = 8.342$
 $\mu - 2\sigma = 4.5 - 2(1.92) = 0.658$
 $P(0.658 < x < 8.342)$
 $= P(0 < x \leq 8)$
 $= P(X \leq 8) - P(X = 0)$
 $= 0.9678$
- c** There is about a 97% chance that from 1 to 8 women in the study group will suffer from iron deficiency.
- 36** $\mu = 12 = np$, $\sigma = 3 = \sqrt{npq}$
 $9 = 12q$
 $q = 0.75$, $p = 0.25$
 $12 = n(0.25)$
 $n = 48$
 $n = 48$, $p = 0.25$, $P(X = 9) = 0.08578$

37 $p = 0.18$ (objects to fluoridisation)

a $n = 10$

i $P(X = 0) = 0.1374$

ii $P(X \geq 2) = 1 - P(x \leq 1) = 0.5608$

b $n = ?$

$$P(x \geq 1) > 0.9$$

$$1 - P(x = 0) > 0.9$$

$$P(x = 0) < 0.1$$

$${}^n C_0 (0.18)^0 (0.82)^n < 0.1$$

i.e. $(0.82)^n < 0.1$

$$n \log_e (0.82) < \log_e (0.1)$$

[Note: $\log_e (0.82) < 0$]

$$n > \frac{\log_e (0.1)}{\log_e (0.82)} = 11.6$$

$$n = 12$$